Teaching the CAPM in the Introductory Finance Course

J. Howard Finch, Steve P. Fraser & Steven R. Scheff

ABSTRACT

The Capital Asset Pricing Model (CAPM) is a core component of introductory financial management courses, capturing the relationship between market risk and expected return. Unfortunately, the interpretation of this relationship is treated differently among financial management textbooks. This paper provides two tools, an equation and a graph, which instructors may use to illustrate how different levels of beta risk translate into expected return. The use of these aids may increase clarity and student understanding of the risk-reward tradeoff inherent in the CAPM.

Introduction

The Capital Asset Pricing Model (CAPM) is a basic component of introductory financial management courses, and has been for many years (Boudreaux and Long, 1970). The model posits a simple linear relationship between a security’s systematic risk exposure, defined by the beta measure, and the expected rate of return. It is an ex ante model which is taught as a method of determining expected (or required) return on equity assets. However, the interpretation of how differing systematic risk exposure translates into differing levels of expected returns is often ambiguous in financial management textbooks, leading to student confusion. In this paper we provide two tools for instructors to help students understand and interpret the risk/reward relationship defined by the CAPM.

Background

Following the literature, we define the CAPM in Equation (1).

\[ E(R_i) = R_F + \beta_i[E(R_{Mkt}) - R_F] \]  

\( E(R_i) \) represents the expected return on asset i; \( R_F \) represents the risk free rate; \( \beta_i \) represents the beta of asset i; and \( E(R_{Mkt}) \) represents the expected return on a market portfolio. Krieger, Fodor and Peterson (2008) note that the CAPM explains risk and return in a way that holds great intuitive appeal and the concept graphs neatly into the Security Market Line (SML). The market is used as the benchmark case, with a beta of 1.0, and other securities with differing systematic risk are compared with the market to illustrate the conceptual risk/reward trade-off.

However, many introductory finance textbooks provide ambiguous statements regarding interpretation of this trade-off. In the analysis below, we illustrate this ambiguity using four widely-used introductory finance texts. Table 1 contrasts the texts’ differing treatments of the interpretation of beta and the effect on expected returns. These excerpts highlight the inconsistency in the portrayal of the risk-reward trade-off captured by the CAPM. Note BHD (2009) states that a company with a beta of 2.0 would be expected to generate more returns, while BB (2008) states that a beta of 2.0 implies a portfolio that doubles in value. GIT (2009) implies twice the responsiveness to market fluctuations results in twice the expected return. Clearly, these statements have different implications regarding how different beta risk translates into different expected returns.
Table 1: Interpreting/Using Beta

<table>
<thead>
<tr>
<th>Text</th>
<th>Page</th>
<th>Summary description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BB</td>
<td>325</td>
<td>If beta=2.0, the stock’s relevant risk is twice as volatile as an average stock, so a portfolio of such stocks will be twice as risky as an average portfolio. The value of such as portfolio could double—or halve—in a short period of time.</td>
</tr>
<tr>
<td>BHD</td>
<td>366</td>
<td>A company with a beta of 2.0 would be twice as volatile as the market and would be expected to generate more returns, whereas a company with a beta of 0.5 would be half as volatile as the market.</td>
</tr>
<tr>
<td>GIT</td>
<td>221</td>
<td>A stock that is twice as responsive to the market (b=2.0) is expected to experience a 2 percent change in return for each 1 percent change in the return of the market portfolio.</td>
</tr>
<tr>
<td>RWJ</td>
<td>351</td>
<td>An asset with a beta of .50 has half as much systematic risk as an average asset; an asset with a beta of 2.0 has twice as much.</td>
</tr>
</tbody>
</table>


A Source of Confusion

In introductory finance texts, the confusion for students in understanding the relationship between a security’s expected return and its beta risk may arise from interpreting the respective slopes of the characteristic line and the security market line (SML). The characteristic line is estimated by regressing the excess returns of the individual security against the excess returns of a market proxy such as the Standard and Poor’s 500 Index to estimate the asset’s beta. Figure 1 depicts an example of the regression resulting in a characteristic line. While the slope coefficient of the characteristic line is the beta, the intercept is approximately zero. A characteristic line with slope of 2.0 implies the security’s excess returns are twice those of the market proxy.

Table 2 summarizes the discussion in each text addressing the estimation or derivation of beta. We find the beta discussion in texts can be found independently, or in conjunction with, the explanation of the CAPM. We find more uniformity amongst the texts in this area.
Table 2: Obtaining/Deriving Beta

<table>
<thead>
<tr>
<th>Text</th>
<th>Page</th>
<th>Summary description</th>
</tr>
</thead>
<tbody>
<tr>
<td>BB</td>
<td>325</td>
<td>Beta is a measure of a stock’s sensitivity to market fluctuations. Beta is the slope of a regression line of the individual returns of a stock on the return of the market.</td>
</tr>
<tr>
<td>BHD</td>
<td>364-6</td>
<td>Beta measures the sensitivity of the security’s return to the market. It is the slope of the regression line of returns between a stock and the market.</td>
</tr>
<tr>
<td>GIT</td>
<td>220</td>
<td>Beta is the slope of the characteristic line that explains the relationship between an asset’s return and the market’s return.</td>
</tr>
<tr>
<td>RWJ</td>
<td>351</td>
<td>Beta tells us how much systematic risk a particular asset has relative to an average asset.</td>
</tr>
</tbody>
</table>


In contrast to a security’s characteristic line, the SML captures the CAPM linear relationship between systematic risk and expected return. Figure 2 depicts the traditional SML. Here, the intercept is not zero, but rather the risk-free rate of return and the slope is the ratio of the expected excess return to beta.

\[
\text{SML: } E(R_i) = R_f + \beta_i (E(R_{Mkt}) - R_f)
\]

Within the CAPM, the equity risk premium (ERP) is defined as

\[
ERP = \beta_i [E(R_{Mkt}) - R_f]
\]  

Students should note that the ERP has two components, the beta coefficient and the market risk premium. The CAPM holds that the expected return on any asset \(i\) depends on both components, not just beta. Many students assume the CAPM to predict that a stock with beta of 2.0 will have an expected return twice that of the market. Yet, because both components of the ERP in the CAPM equation contribute to expected return, changes in beta risk are not proportional to changes in expected return.
Clarifying the Risk-Expected Return Relationship

We examine the effect of the components by taking the derivative of the CAPM shown in Equation (1) with respect to beta. To illustrate the effect of a beta different from that of the market on expected return, let the base case be the market beta of 1.0 where \( E(R_i) = E(R_{Mkt}) \), and assume, ceteris paribus, that beta changes.

\[
\frac{dE(R_i)}{d\beta_i} = E(R_{Mkt}) - R_F
\]

The left-hand side of Equation (2) gives the percentage change in expected return for a change in beta. Further simplification of the right-hand side results in Equation (3):

\[
\%\Delta \ln E(R_i) = (\beta_i - \beta_{Mkt}) \left[ 1 - \frac{R_F}{R_{Mkt}} \right]
\]

Equation (3) generalizes the relationship for how expected return for an individual asset deviates from that of the expected market return.

Applying Equation (3) to specific examples will highlight the issue of the sensitivity of expected return to differences in the beta coefficient. Consider an economy where the \( R_{Mkt} = 15 \) percent and the \( R_F = 5 \) percent. The expected return for a stock with beta of 1.0 is given by:

\[
E(R_i) = 0.05 + 1[0.15 - 0.05] = 0.15 \text{ or 15%}
\]

How much will the expected return increase for a stock with double the market risk (beta = 2)? Applying Equation (3),

\[
\%\Delta \ln E(R_i) = (2 - 1)[1 - \frac{0.05}{0.15}] = 0.66 \text{ or 66%}
\]

Thus, under these assumptions a stock with beta of 2 will have 66 percent greater expected return than a stock with beta of 1. This result is confirmed by applying the CAPM directly for a stock with beta of 2.0.

\[
E(R_i) = 0.05 + 2[0.15 - 0.05] = 0.25 \text{ or 25%}
\]
Here, beta risk increases by 100 percent (beta changes from 1.0 to 2.0) while the expected return increases only 66 percent (from 15 to 25 percent). Students can see that twice the systematic risk does not result in twice the expected return. The key insight is the relationship of the risk-free rate to the expected market return captured in the term on the right-hand side of Equation (3). The equation allows instructors to illustrate the contribution of the prevailing market risk premium on the CAPM expected under differing market conditions.

While many finance texts imply double the beta results in double the expected return, this example demonstrates that the change in expected return was only 66 percent of the change in beta. The reason is apparent when the benchmark case is examined in closer detail. With a beta of 1.0, the required rate of return predicted by the CAPM is 15 percent. However, one-third (5 percent) of this return is risk free, which is not a function of the individual security’s market risk. Only the market risk premium, the return in excess of the risk free rate, is influenced by the beta coefficient. In this case, only two-thirds of the asset’s total expected return comes from the equity risk premium. Extending the base case example of \( E(R_{Mkt}) = 15 \) percent and \( R_F = 5 \) percent, Table 3 details the percentage contributions of each component of expected return predicted by the CAPM for assets with betas different from that of the market.

### Table 3: Decomposition of CAPM Expected Returns

<table>
<thead>
<tr>
<th>Beta</th>
<th>CAPM Expected Return</th>
<th>% Contribution from the Risk-Free rate</th>
<th>% Contribution from the Risk-Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.00</td>
<td>5.00</td>
<td>100%</td>
<td>0%</td>
</tr>
<tr>
<td>0.25</td>
<td>7.50</td>
<td>66%</td>
<td>33%</td>
</tr>
<tr>
<td>0.50</td>
<td>10.00</td>
<td>50%</td>
<td>50%</td>
</tr>
<tr>
<td>0.75</td>
<td>12.50</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>1.00</td>
<td>15.00</td>
<td>33%</td>
<td>66%</td>
</tr>
<tr>
<td>1.25</td>
<td>17.50</td>
<td>29%</td>
<td>71%</td>
</tr>
<tr>
<td>1.50</td>
<td>20.00</td>
<td>25%</td>
<td>75%</td>
</tr>
<tr>
<td>1.75</td>
<td>22.50</td>
<td>22%</td>
<td>78%</td>
</tr>
<tr>
<td>2.00</td>
<td>25.00</td>
<td>20%</td>
<td>80%</td>
</tr>
</tbody>
</table>

Note - Base case with Risk-Free rate=5% and Expected Return of the Market Portfolio=15%

For a security with a zero beta, the equity risk premium contributes nothing to the expected return. In contrast, greater market risk translates into greater expected return. The equity risk premium contributes up to 80 percent of expected return when beta is 2. Figure 3 allows students to see that the contribution of the equity risk premium increases with beta risk; however, it does so at a decreasing rate.

![Figure 3: Expected Return Contribution](image)
This example illustrates that assets with beta risks different from the market benchmark beta of 1.0 will have different expected returns, but the assumption that changes in beta risk result in proportional changes in expected return may mislead students regarding the risk reward trade-off. Both Table 3 and Figure 3 hold the assumptions of the market factors constant. Instructors may assign students the task of recreating the table and then the graph under different market conditions to illustrate how shifts in uncertainty and interest rates translate into different equity risk premiums, and the resulting contributions to expected return using the CAPM.

Conclusion

This paper provides two contributions to the classroom explanation of the CAPM in introductory finance courses. First, to examine the contribution of the systematic risk component to the expected return predicted by the CAPM, we provide a straightforward equation using the same variables from the CAPM that allows students to calculate the percentage change in expected return that results from beta risk different than the market. Instructors may use the equation to illustrate that, as assumptions regarding the risk-free rate and expected market return change, the sensitivity of the asset’s expected return to beta risk will change as well. Second, we graphically illustrate the respective contributions of the components of the model’s expected return. Instructors can use the graph to highlight the non-linear relationship of the contribution of the equity risk premium to expected return. Using these tools instructors may conduct scenario analyses in class that highlights the sensitivity of expected asset return to changes in beta risk. This process may reduce confusion and help students understand the critical risk reward trade-off implied by the Capital Asset Pricing Model.

References


