

# Enhancing Clarity and Completeness of Basic Financial Text Treatments on Operating Leverage

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## Abstract

We review degree of operating leverage (DOL) discussions from nine current elementary finance textbooks and note incomplete treatments. We argue that textual discussions on DOL could aid the beginning student's understanding by noting, in addition to fixed costs, the impact of other variables (unit variable costs, unit price, short-run output level). We further suggest that complete treatments should reference how the DOL measure fits into the larger business risk context, point to management's role in influencing selected DOL variables as business risk parameters, and mention the measurement discontinuities of DOL at breakeven and profit maximizing output levels.

## Introduction

Basic finance text authors display a variety of algebraic approaches to but address only selected aspects of operating leverage and its use by firm management. No basic finance text DOL treatment appears complete. Most discussions ignore the role of other variables in the DOL measure and its inherent mathematical discontinuities. This article reviews those unstated aspects of the measure and offers a more complete framework for operating leverage text discussions aimed at elementary finance students.

Operating leverage is important to firm management for one reason, additions to operating fixed costs affect a firm's value by increasing risk, where risk is measured by the variability of returns (Lev, 1974, and Berner, 2002). Operating leverage changes alter the firm's business risk position. Text authors often introduce the degree of operating (DOL) measure as a natural extension to linear breakeven analysis and some use it as a logical segue into risk and return discussions.

Although DOL equation structures of choice vary among text writers, they often present DOL as the percentage change in operating profit relative to a percentage change in output, a profit elasticity form of the measure. This form is mathematically equivalent to each of the alternatives that appear in basic finance texts. Aspects of DOL we consider important to cover are: a) to a look at the larger business risk context within which the DOL measure is relevant, b) a clearer view of management's role in influencing certain DOL parameters as business risk components and c) some rather subtle but important limitations inherent in the measure itself.

Business risk is a central determinant of a firm's value, the risk-adjusted present value of future profit. Several important parameters affect a firm's business risk position. Among them are price, variable costs, operating fixed costs, the stability of demand and the output rate. The DOL measure contains variables that capture four of these parameters. Demand stability is a through-time assessment while DOL is a point-in-time measure. The level of operating fixed cost, the parameter of greatest attention in textual DOL discussions, is only one business risk parameter.

Changes in the magnitude of any variable in the DOL expression alter the magnitude of the result. A change in a single business risk parameter in the DOL expression can also affect the remaining parameters. For example, increases in operating fixed cost without a compensating reduction in unit variable cost may

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require increases in output to sustain a desired profit level. A meaningful discussion of DOL should at least mention the distinction between management-led choices addressing the firm's business risk posture versus market forces or engineering-based relationships. Finally, measurement orientation and mathematical discontinuities inherent to the DOL measure deserve mention.

Discussion in Section 2 confirms the mathematical equivalence between various DOL measures and discusses articles by Dran (1991), Long (1992), Dugan et al. (1994) and O'Brien and Vanderheiden (1987) to show that DOL is sensitive not only to changes in the firm's operating fixed cost but also to short run output. That section suggests that narrative treatments indicating which DOL parameters management can directly influence would help place the measure into a useful operational context. Section 3 extends the DOL expression to include a non-linear (parabolic and upward sloping) variable (and total) cost function to demonstrate that DOL equals zero at the theoretically optimized output, regardless of the level of fixed cost. Section 4 narrative centers on how completely each of nine basic finance text authors treats the DOL concept and measure. Section 5 provides the authors' suggestions on how to coalesce the DOL treatment in textual narrations, given the arguments made.

### Theoretical Review

As derived from short run linear revenue and cost functions for the single product, profit maximizing competitive firm under certainty, the DOL measure is essentially a profit-cum-output sensitivity ratio. The textbook DOL expression takes one of several forms. The list below suggests the variety of equivalent DOL equations used in basic finance texts.

$$\begin{aligned} \text{DOL} &= (\% \text{ Change in } \Pi) / (\% \text{ Change in } Q) = Q(p-v) / [Q(p-v) - FC] = \\ &= (\text{TR} - \text{TVC}) / (\text{TR} - \text{TVC} - \text{FC}) = (\Pi + \text{FC}) / \Pi \end{aligned} \tag{1}$$

For convenience and from the development in the Appendix, these mathematically equivalent DOL expressions above reduce to:

$$\text{DOL} = 1 + \text{FC}/(p \cdot Q - v \cdot Q - \text{FC}) \tag{2}$$

where;

FC = operating fixed cost

P = unit price

v = unit variable cost

Q = quantity of output

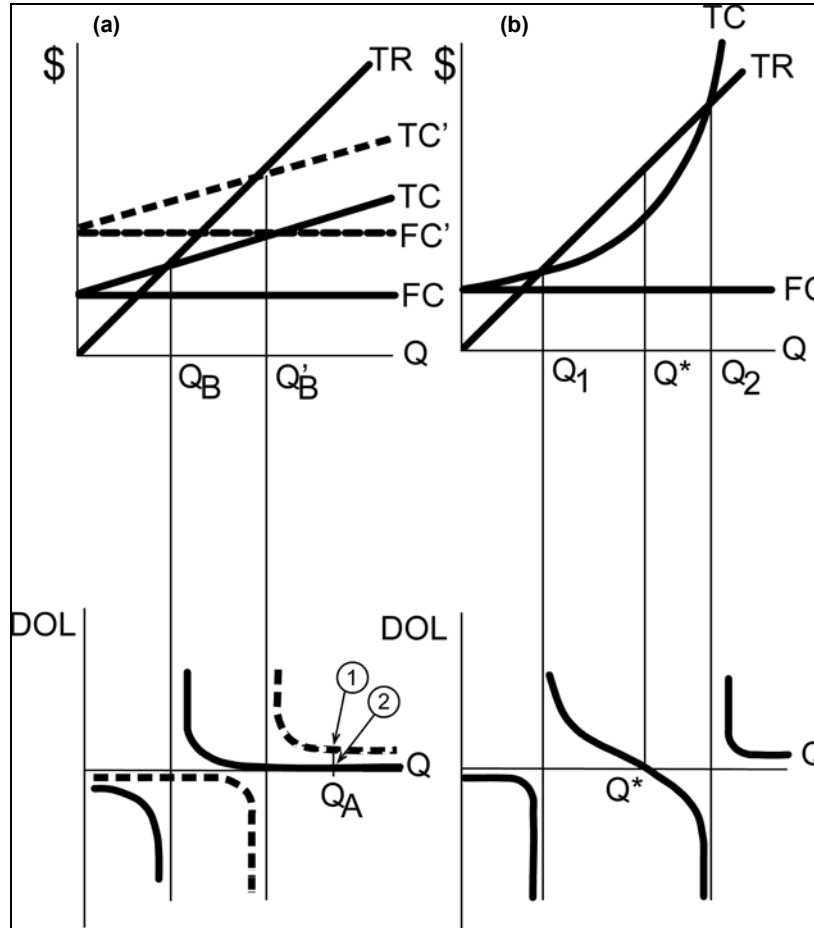
Π = earnings before interest and taxes = EBIT = p·Q – v·Q – FC

One could say that the profit sensitivity model for DOL used by finance text authors is a stylized version suitable for pedagogical exposition. To the extent that this may be true, it is important for the beginning student's understanding that authors provide more, rather than less, thorough discussions regarding DOL. While alternatives to the profit sensitivity form to measure DOL exist for empirical work, there appears to be no consensus among empiricists or theorists which business parameter provides overall consistent and superior operating leverage measurement results. Dugan (1994) and O'Brien (1987) observed that alternative measures of operating leverage altered their empirical findings as did the choice of statistical technique. O'Brien also suggested that non-linearities O'Brien (1987, p. 50) could account for much of the variation in their approach to measure operating leverage for growing firms.

Dran (1991) and Long (1992) provided finance literature a theoretical treatment that demonstrated how proximity to breakeven output influences DOL, independent of the level of operating fixed cost. Dran did so by defining the firm's output as a percentage of breakeven quantity. By separating the traditional DOL measure from the firm's cost structure it became mathematically obvious that DOL was also sensitive to the firm's output level, rising or falling asymptotically toward positive or negative infinity as breakeven output was approached either from above or below.

In a reply to Dran's original contribution, Long (1992) showed that there was not a priori fixed-in-proportion economic relationship between increases in fixed operating expenses and commensurate reductions in unit variable cost sufficient to maintain the prior breakeven output. For this to be so, the

**Figure 1**  
**Breakeven and Operating Leverage**  
**Linear (a) and Non-linear Cases (b)**



revised unit variable cost would have to fall sufficiently to compensate for the increased operating fixed cost in both the denominator and the numerator to leave the ratio value in Equation (2) unchanged after an increase in operating fixed cost.

While there logically exists a lower unit variable cost value that could compensate for increased operating fixed costs sufficient to leave both breakeven output and DOL unchanged, engineering and economic relationships determine that value more than management. Empirical investigators (Lord, 1995, and Li, 1991) offer evidence supporting that management recognizes and considers such a tradeoff. Unit variable costs may fall more or less than proportionately to compensate for the increased operating fixed cost, due to engineering relationships associated with new capital infusion, thus requiring a decreased or increased output level to breakeven.

From Equation (2) above, if  $FC = 0$ , then  $DOL = 1$ , indicating that there is no operating leverage. As FC assumes any positive value and, for simplicity, if profit remains positive and there is no change in unit variable cost, then DOL must rise above 1 for two analytical reasons. With an increased operating FC level, if price and unit variable cost remain the same, the denominator in the second part of the expression is smaller and the numerator is larger.

This is precisely the point where several textual treatments begin to get murky by confusing what is mathematically possible with what is economically feasible. Logically, a firm’s management seeking to maximize profits will not voluntarily permit operating FC to rise without a commensurate fall in unit variable costs or a possible increase in price, if price-setting is within their power and strategically desired. In a competitive market, price setting simply is not an option. The DOL parameters are economically interdependent.

Given the assumed linear revenue and cost functions and regardless of the operating fixed cost level, the closer operating profit gets to zero, DOL approaches negative or positive infinity in the vicinity of breakeven quantity, depending on whether the breakeven output is approached from below or above. At the breakeven quantity, DOL provides no useful value. For output levels above the breakeven quantity, DOL falls asymptotically toward zero as output increases because profit in the denominator continues to increase while operating fixed cost is constant in the numerator. So, DOL varies for two reasons: the level of operating fixed cost and the output, both of which management determines.

Readings of the DOL result are meaningful only when compared to the same output level. Looking at Figure 1(a) and output level  $Q_A$  clearly shows the result of a higher FC on the DOL at that output level. The measured DOL, (1), after the increase in FC is clearly greater than the originally measured DOL, (2). Depending on the prior output level, even a small increase in FC, depending on the slope of the TR function, with no change in unit variable costs for simplicity, would require a management decision to increase output to avoid losses to maintain desired profits. Failure to do so could lead to short run operating losses. This is shown in Figure 1(a) by comparing breakeven quantities  $Q_B$  and  $Q_{B'}$  that differ only due to a greater FC.

Even in a perfectly competitive market, price plays a role in determining the DOL magnitude. Assume a firm is currently operating with a positive profit. When market equilibrium price rises, the denominator in Equation (2) rises reducing the DOL with no change in output, operating fixed cost, or unit variable cost. Consequently, breakeven output falls since the contribution margin is larger. DOL can vary due to changes in any of the variables appearing in Equation (2). These variables include management-determined choices—operating fixed cost and output levels; market determined parameters—price in a competitive market as time passes; and economic and engineering realities—unit variable costs, given operating fixed cost increases due to new capital acquisition.

### **A Theoretical Extension**

A theoretical extension provides additional insight into DOL measurement limits. Relax the assumption of a linear total variable cost function in favor of a twice-differentiable and increasing parabolic function. The resulting profit elasticity expression from the development in the appendix appears below.

$$E_{\pi} = [(p - MC) Q] / [(p - AVC) Q - FC] \tag{3}$$

It is clear from Equation (3) that DOL must be zero when profit is maximized or when losses are minimized, because at that point a positive  $p$  must equal a negative  $MC$  in the numerator. It is also clear that as  $p$  approaches  $AVC$ , or equivalently,  $TR$  approaches  $TC$ , then DOL will approach positive or negative infinity depending on the direction from which quantity approaches breakeven. These analytical results are true, regardless of the level of fixed cost. Hence, there exists two points where the DOL measure provides no useful information: breakeven quantity and profit maximizing/loss minimizing quantity.

Figure 1 shows the ranges of DOL values, given linear 1(a) and curvilinear 1(b) cost functions. Notice in Figure 1(a) when fixed costs rise from  $FC$  to  $FC'$ , with no compensatory reduction in unit variable costs, two things occur. First, the breakeven quantity of output rises. Second, the DOL magnitude for any quantity above the new breakeven point is greater than before the addition of fixed cost. This is precisely what the DOL expression should show as a firm’s business risk indicator. We simply note that DOL will vary due to changes in other of its parameters that management controls, most notably output.

Figure 1(b) reveals the influence on DOL from a more general curvilinear cost function, other assumptions the same. Just as in the linear case, DOL approaches infinity at the breakeven quantity levels of output. At the profit maximizing output level, DOL is zero. For firms with rising upward sloping costs DOL, in general, equals zero at the optimizing level of output, given any level of fixed cost. Such a result

Table 1. Textual treatment of operating leverage and definitions

<i>Authors</i>	<i>Operating Leverage</i>	<i>DOL</i>	<i>Implications</i>	<i>How to measure DOL</i>
Brigham and Houston (2002)	The extent to which operating fixed costs are used in a firm's operation	If a high percentage of total costs are fixed, then the firm is said to have a high degree of operating leverage	A high degree of operating leverage implies that a relatively small change in sales results in a large change in ROE	N/A
Ross, Westerfield, and Jordan (1998)	The degree to which a project or firm committed to fixed production cost	The percentage change in operating cash flow relative to the percentage change in quantity sold	A small percentage change in operating revenue can be magnified into a large percentage change in operating cash flow and NPV	$DOL = 1 + \frac{FC}{OCF}$
Van Horne and Wachowicz (2001)	The use of fixed operating cost by the firm	The percentage change in firm's operating profit (EBIT) resulting from a 1 percentage change in output (sales)	Potential effect of operating leverage is that a change in the volume of sales result in a more than proportional change in operating profit (or loss)	$DOL = \frac{\% \Delta \text{ in EBIT}}{\% \Delta \text{ in output}}$  $= \frac{EBIT + FC}{EBIT}$
Lasher (1997)	Operating leverage refers to the amount of operating fixed cost in the cost structure	DOL is the ratio of the relative change in EBIT to a relative change in sales	Amplifies changes in sales volume into larger changes in EBIT	$DOL = \frac{\% \Delta \text{ in EBIT}}{\% \Delta Q}$ $DOL = \frac{Q(P-V)}{Q(P-V) - FC}$
Gallagher and Andrew (2000)	The phenomena whereby a small change in sales triggers a relatively large change in operating income	DOL is the percentage change in EBIT divided by a percentage change in sales	A firm w/operating fixed costs in the production process will see its EBIT rise by a larger percentage than sales when unit sales increases	$DOL = \frac{\% \Delta \text{ in EBIT}}{\% \Delta \text{ in sales}}$
Brealey, Myers, and Marcus (2004)	Degree to which costs are fixed	Percentage change in profits given a 1 percent change in sales	High operating leverage magnifies the effect on profits of a fluctuation in sales	$DOL = \frac{\% \Delta \text{ in profit}}{\% \Delta \text{ in sales}}$
Block and Hirt	The extent fixed assets (plant and equipment) are	Percentage change in operating income that occurs as a result of a	By increasing leverage, the firm increases its profit	$DOL = \frac{\% \Delta \text{ in Op. Income}}$

(2002)	utilized by the firm	percentage change in units sold	potential, but also increases its risk of failure	$\% \Delta$ in unit vol.
Keown, Martin, Petty, and Scott (2003)	The responsiveness of firm's EBIT to fluctuations in sales	Percentage change in EBIT given percentage change in sales	The greater the firm's degree of operating leverage, the more its profits will vary with a given percentage change in sales	$DOL = \frac{Q(P-V)}{Q(P-V) - FC}$
Melicher and Norton (2003)	Sensitivity of operating income to changes in level of output	Percent change in EBIT resulted from percentage change in sales	A higher level of fixed cost results in a higher level of operating leverage	$DOL = \frac{\% \Delta \text{ in EBIT}}{\% \Delta \text{ Sales}}$

poses a conflict of sorts between an important concept in economic theory and an important risk measure in finance theory. DOL for the theoretically optimal output in economics becomes impossible to measure directly.

### Basic Finance Text Treatments of Operating Leverage

Introductory finance textbook treatments uniformly incorporate linear cost and revenue functions to motivate discussions on breakeven analysis. Applied linear breakeven analysis serves as a first approximation to real-life business settings. Given that the hurdle to be exceeded by the margin of unit price over unit variable cost times unit volume in breakeven analysis is operating fixed cost, textual narrative regarding the degree of operating leverage follows in many instances.

All nine textbooks reviewed equate operating leverage with the level of operating fixed cost in a firm's operation. The degree of operating leverage, DOL, is most widely defined as the sensitivity of earnings (i.e. EBIT) to changes in sales revenue or quantity. This interpretation of DOL is based on the notion that, in the presence of operating fixed costs, a small percentage change in sales may result in a larger percentage change in earnings—greater business risk, something about which corporate financial stewards should be aware.

As with all indicators, DOL manifests useful characteristics and limitations. Its usefulness centers on its simplicity. As operating fixed costs rise, the DOL magnitude typically will rise. Its limitations relate to its sensitivity to changes in its other parameters and with measurement discontinuities. A change in the magnitude of any variable in the DOL expression, including quantity, results in an altered DOL magnitude. Only two of nine text writers in our search directly mention this fact.

Van Horne and Wachowicz (2002) and Block and Hirt (2002) note that variables other than operating fixed cost affect DOL. Van Horne and Wachowicz correctly speak to how a firm's output proximity to its breakeven output, not just the absolute or relative amount of fixed operating cost, determines the DOL measure's magnitude. Block and Hirt place this point in a footnote. Block and Hirt are the only ones who directly hint at the possible effects of non-linear cost functions (Block and Hirt, 2002, p. 118).

Each of the nine popular basic finance texts written for the college and university market introduces the degree of operating leverage measure in algebraic, graphical or elasticity form, or some combination of these forms, after discussing breakeven analysis.<sup>1</sup> One set of authors (Brigham and Houston, 2002) omits the equation itself. Text authors, apparently for simplicity and instructive purposes, assume linear revenue and cost relationships to motivate the DOL discussion.

Other textual discussions imply that positive net present value options to acquire new capital, while increasing fixed operating costs may also reduce unit variable cost as a trade-off benefit, but pay little

<sup>1</sup> We also reviewed five intermediate and graduate level books. Most of these textbooks cover DOL in much the same way as basic finance textbooks.

attention to the fact that the resulting breakeven output may rise, fall or remain the same. It is not necessarily true that a given increase in operating fixed costs, due perhaps to new technology introduction, will automatically reduce variable unit costs sufficiently to maintain the original breakeven output.

Elementary finance students draw the lesson that increasing operating fixed costs in the firm's operating cost structure adds to business risk. The lesson seems obvious and perhaps that is sufficient introduction at the elementary level. The higher the operating fixed cost hurdle for a firm in the short run, the smaller the chance that the margin of unit price above unit variable cost times the count of units sold will be sufficient to generate a profit. Our view is that more framing and acknowledgement of limits would better serve the learner with a relatively small commitment of valuable page space.

### A Suggested Revision

We recommend a more complete framing and discussion on DOL limitations to form a more cohesive picture in the student's mind. We emphasize that the changes in DOL that are most useful are those showing the expected consequences of management-led decisions on the DOL magnitude *at a given level of output*. It is the change in DOL, output constant, brought about by management's decisions, where DOL is most compelling as a business risk measure.

Table 2. DOL Parameters and Business Risk Dimensions

<i>DOL Variable</i>	<i>Determined by:</i>
Unit Price (P)	<u>Market Forces:</u> Competitive market
	<u>Management Decision:</u> Non-competitive market
Unit Variable Cost (VC)	<u>Engineering and Economic relationships:</u> All markets
Operating Fixed cost (FC)- Long run	<u>Management Decision:</u> All markets
Output (Q)- Short run	<u>Management Decision:</u> All markets

We separate the DOL expression variables into three related categories: management decision variables, market-determined variables, and engineering variables. Management must assess the effect of any change in operating fixed cost on unit variable costs prior to committing to the decision. Unit variable cost is partly determined by the market, i.e. factor inputs at their market rate per time and partly by the engineering relationships that exist between old versus new capital equipment and related labor support requirements. The relationship between fixed cost changes and unit variable cost changes helps determine the new required minimum output for the firm to breakeven in the short run.

As firm management evaluates production cost reduction strategies, they have three options: increase production efficiency, outsource an operating fixed cost component to make it a variable cost, or acquire new technology that reduces unit variable cost as long as it does not increase operating fixed costs too greatly. Once management acts on the commitment to increase operating fixed cost, reversing the decision is neither easy nor quick, giving it long run implications.

To place algebraic or graphical expositions on DOL into a fuller context, the essential points to emphasize in basic finance textual discussions on DOL reduce to the following:

- The DOL measure contains several parameters affecting a firm's business risk. Operating fixed cost is only one parameter.
- The effect of DOL changes from an increase in operating fixed cost should be observed relative to the same output level.
- DOL as a magnitude also varies with changes in output in the short run, a management decision.
- DOL provides no useful information for firms operating at or very near their operating breakeven output level or their profit maximizing (or loss minimizing) output level.
- Engineering relationships largely dictate the relation between changes in operating fixed cost and unit variable cost; and management must assess each case on its own merit.
- Whether or not management influenced the change, DOL varies with a change in magnitude of any DOL parameter.
- Whether and how much influence over price management has depends on the competitiveness of the product's market.

### **Summary and Conclusion**

A review of operating leverage discussions from a selection of nine basic finance textbooks reveals that relevant aspects of DOL are absent from many textual discussions. We recommend that useful aspects include: a) reference to the larger business risk context within which the DOL measure resides, b) a clearer view of management's role in influencing DOL parameters as business risk components and c) mention of some rather subtle but important limitations inherent in the measure itself.

Dran and Long, using the profit sensitivity version of DOL, demonstrate the dual influence on profit sensitivity from operating fixed cost changes as well as from output changes. Each of the other variables in the DOL expression also can affect the DOL magnitude. We suggest the usefulness of separating DOL parameters into those that management can influence, those that the market influences and those determined mostly by engineering relationships.

By changing only one assumption used in elementary models of the firm, from linear to non-linear variable cost functions, keeping other assumptions intact -single product, short run, certainty and linear functions- it was shown that DOL equals zero when the firm's output is optimized, regardless of the level of operating fixed cost. Hence, two measurement discontinuities for DOL exist, the quantity breakeven output and the profit maximizing or loss minimizing output.

We further suggest that including the following points in DOL narrative discussions would enhance a beginning finance student's understanding of the larger business risk context, sources of all formulaic DOL variability and more about its measurement limitations.

A more complete DOL discussion would include the following points: a) the DOL profit sensitivity expression contains several business risk parameters, b) DOL changes due to operating fixed cost changes should be measured at the same output level, c) DOL varies with changes in output in the short run, a management decision, d) DOL provides no useful information at the firm's operating breakeven output level or profit maximizing (loss minimizing) output level, e) engineering relationships largely affect the relation between changes in operating fixed cost and unit variable cost and must be weighed a priority by management in each case, f) DOL numerically changes with a change in magnitude of any DOL parameter, and h) whether management has influence over price depends on the competitive nature of the product's market.



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### Appendix

Assume a single product firm in the short run under certainty operating in competitive input and output markets. Total revenue and total cost functions are linear. The degree of operating leverage (DOL) function in standard textbook profit elasticity form is commonly stated as:

$$\begin{aligned} \text{DOL} &= \% \Delta \Pi / \% \Delta Q & (1) \\ &= [(\Delta p \cdot Q - \Delta v \cdot Q - \Delta FC) / (p \cdot Q - v \cdot Q - FC)] [Q / \Delta Q] \\ &\text{where } \Delta FC = 0 \\ &= [(p \cdot Q - v \cdot Q) / (p \cdot Q - v \cdot Q - FC)] \end{aligned}$$

$$\text{DOL} = Q(p-v) / [Q(p-v) - FC] \quad (2)$$

$$\text{DOL} = (TR - TVC) / (TR - TVC - FC), \text{ where} \quad (3)$$

Q = quantity output per time  
 p = selling price per unit of output  
 v = variable cost per unit of output  
 FC = total operating fixed cost  
 TR = total revenue  
 TVC = total variable cost  
 Π = profit.

Financial and economic analysis defines Π as,

$$\begin{aligned} \Pi &= TR - TVC - FC & (4) \\ \Pi &= p \cdot Q - v \cdot Q - FC. \end{aligned}$$

If profit = 0, then

$$Q = FC / (p - v), \text{ to solve for breakeven quantity of output.} \quad (5)$$

For any Π value other than zero,

$$\begin{aligned} \Pi - TR + TVC &= -FC \\ TR - TVC &= FC + \Pi \end{aligned} \quad (6)$$

Substituting (4) and (6) into (3) gives

$$\text{DOL} = (\Pi + FC) / \Pi, \text{ or} \quad (7)$$

$$\text{DOL} = 1 + (FC / \Pi) = 1 + FC / (p \cdot Q - v \cdot Q - FC) \quad (8)$$

and the DOL becomes discontinuous at Π = 0.

Now assume a quadratic and continuously positively sloped total variable cost function that is twice differentiable. Standard economic optimization theory confirms profit maximization as follows:

$$\Pi = TR(Q) - TC(Q) \quad (9)$$

Take the first derivative as the necessary condition, to determine candidate values for Q,

$$\Pi' = R'(Q) - C'(Q) = 0, \quad (10)$$

and the second derivative sufficient condition to test the candidate values from (10)

$$\Pi''(Q) = R''(Q) - C''(Q) < 0. \quad (11)$$

The logical progression now closely parallels Rives' development.

$$TR = p \cdot Q, \text{ where } p > 0 \text{ and constant} \quad (12)$$

$$TC = a + b \cdot Q + c \cdot Q^2 \quad (13)$$

Where  $a = FC$ ,  $b \cdot Q + c \cdot Q^2 = TVC$ , and profit equals

$$\Pi = TR - TC = (p \cdot Q) - (a + b \cdot Q + c \cdot Q^2) \quad (14)$$

$$\Pi = (p - b - c \cdot Q) Q - a$$

$$d\Pi/dQ = p - b - 2c \cdot Q \quad (15)$$

$$MR = dTR/dQ = p \quad (16)$$

$$MC = dTC/dQ = b + 2c \cdot Q \quad (17)$$

From (1) above and in continuous change notation, the quantity elasticity of profit, DOL, can be written as:

$$E_{\Pi} = (d\Pi/dQ) \cdot (Q/\Pi). \quad (18)$$

Substituting and inserting common economic variable names gives:

$$E_{\Pi} = [p - (b + 2c \cdot Q)] Q / [(p - b + c \cdot Q) Q - a]$$

$$E_{\Pi} = [(p - MC) Q] / [(p - AVC) Q - FC]. \quad (19)$$

Equation (19) is the formal algebraic equivalent DOL expression in a profit elasticity structure, though with key variables expressed in familiar economic terms. To the extent that MC can be estimated with some precision, there need no longer be an issue over the chosen base from which to measure percentage changes using expression (19) to assess DOL. It is now also clear that when output is at the profit maximizing level, DOL must be zero, since  $p = MC$  in the numerator. Further, it is true that DOL equals zero if the firm is minimizing losses for the same reason.