# The Pricing of Bonds between Coupon Payments: From Theory to Market Practice

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## ABSTRACT

The transition for students from theory to practice is often abrupt and awkward. An example is the pricing of a bond that is between coupon payments. In this situation, students of finance may benefit from further exploration of the effect of compound interest before learning of the method that financial markets actually use to determine the price an investor must pay. This paper explores the confusing link between theoretical pricing of coupon bonds and the practice used by financial markets. Common financial calculators are used as a bridge to highlight this connection.

## Introduction

The fair market price of a coupon bond is determined by discounting both an ordinary annuity and a future lump sum. Students in the typical introductory finance course learn these techniques shortly after time value of money fundamentals have been presented. In most upper level finance courses, students learn how the financial markets actually determine the transaction price that an investor must pay to purchase a coupon bond. It is essential in upper level courses that both the instructor and the textbook address the pricing of a bond between coupon payments. Frequently, it is an awkward transition to market practice that confuses students of finance because (1) market practice is not consistent with the principles of compound interest, and (2) the media does not quote the actual transaction price that an investor must pay for a coupon bond.

Many finance texts imply that the price quoted by the financial media is the price that an investor must pay to obtain a particular bond. A typical example is how Brealey et al. (2007, pp. 118-128) begin a discussion of bond valuation. An excerpt from the *Wall Street Journal* containing market quotes of U.S. Treasury Bonds and Notes is provided, and the text specifically states:

> *The prices at which you can buy and sell bonds are shown each day in the financial press.*

After discussing the difference between bid and ask prices, Brealey et al. conclude that, "*Therefore each bond costs $1,057.19.*" Presenting bond valuation in this fashion leads the student to believe that the price quoted by the financial media is the actual transaction price that occurs between buyer and seller.

Consider the confusion on the part of the student enrolled in a later finance course that uses Bodie et al. (2005). The text states on page 450:

> *The bond prices that you see quoted in the financial pages are not actually the prices that investors pay for the bond. This is because the quoted price does not include the interest that accrues between coupon payment dates.*

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By quoting the price of a bond without accrued interest, financial markets avoid drastic changes in price that would occur on or following coupon payment dates. Bodie et al. (2005) then briefly explain that the transaction price of a bond between coupon payments consists of the quoted price plus accrued interest since the prior coupon payment. Francis and Ibbotson (2002, p. 573) note the difference and also add accrued interest to a quoted price to determine the transaction price. Students enrolled in a course that uses Ross et al. (2006, p. 219) will learn that the quoted price is obtained from the transaction price *less* the interest that *will* accrue until the next coupon payment.

Brigham and Daves' (2007, p. 123) treatment of the pricing topic is found within their description of spreadsheet solutions to bond valuation and consists of the sentence, *"This (*spreadsheet*) function is essential if a bond is being evaluated between coupon payment dates."* Bodie et al. (2005) also rely solely on the use of pre-programmed spreadsheet functions to determine the transaction price of a bond between coupon payments.

Saunders and Cornett (2004, pp. 159-160) introduce the concept of finding the transaction price that a buyer will pay for a bond between coupon payments by assuming that the quoted price is known. The example provided is a simple matter of adding discretely determined accrued interest since the prior coupon to the quoted price of the bond. Corrado and Jordan (2005, pp. 330-332) use an opposite approach and briefly explain how the quoted price is determined. They assume the transaction price is known and a discretely determined amount of accrued interest is deducted to arrive at the quoted price.

Fabozzi (2004, pp. 138-141) provides an explanation of how the market determines the transaction price and identifies the method used by the markets to find a quoted price. However, the text is specifically written for the Chartered Financial Analyst® Program and is not in the academic mainstream. Hearth and Zaima (2001, pp. 213-214) use virtually the same market formula as Fabozzi (2004) for the transaction price but the example provided is not consistent with their formula. Fabozzi & Fabozzi (1989, pp. 27-31) explicitly state the formula that the market uses to determine the price of a coupon bond, but do not explain that it will lead to the transaction price rather than the quoted price.

Of the academic texts reviewed, none specifically and/or comprehensively explain that financial markets *first* determine a theoretically correct transaction price for a coupon bond, and *then* subtract discretely determined accrued interest to arrive at the quoted price used by the financial media. Further, the link between discretely determined interest and accepted theories of compound interest is left unexplored.

This paper presents examples that suggest a format instructors may use to facilitate students' transition between the theoretical pricing of coupon bonds and actual market practice. First, market practice is explored and a continuing example is introduced. Second, the departure from theory that financial markets make to determine a price quoted by the financial media is explained. Finally, the basic fair market price formula is extended to include the use of daily compounding of interest to price a bond that is between coupon payments.

The exercise presented here builds on bond pricing fundamentals understood by students and should be a relatively natural progression. In a few steps, students can determine the transaction price an investor should be willing to pay for a coupon bond on any given day between two coupon payments.

## The Market-Determined Transaction Price of a Bond between Coupon Payments

As is often the case, textbook solutions are not easily applied in practice. According to Saunders and Cornett (2004, pp. 158-160), the financial media will quote the price of the bond without accrued interest. The transaction price that an investor will actually pay must be calculated separately and depends on the coupon rate and the number of days since the prior coupon payment. Assuming that a quoted price is available on any given day, Saunders and Cornett (2004) use Equation (1) to determine the accrued interest that the seller of a bond will demand when disposing of the security:

$$(1) \quad AccruedInterest = \left[\frac{AnnualCouponPayment}{2}\right] X \left[\frac{DaysSinceLastCoupon}{DaysInCouponPeriod}\right]$$

This form indicates that there is a discrete amount of interest added to the current quoted price of the security and the method is confirmed by Hearth and Zaima (2001), Francis and Ibbotson (2002) and Fabozzi (1996). Saunders and Cornett (2004) however, do not explore how the market arrives at the quoted price of the security. Fabozzi and Fabozzi (1989, p. 29) and Hearth and Zaima (2001, p. 214) both identify the formula used in market practice to determine the transaction price of a bond between coupon payments. The bond pricing formula that Fabozzi & Fabozzi (1989, p. 29) claims the "Street" uses to compute the transaction price of a bond is:

$$(2)\quad P_{transaction} = \sum_{t=1}^{n} \frac{C}{(1+r)^{v}(1+r)^{t-1}} + \frac{M}{(1+r)^{v}(1+r)^{n-1}}$$

Where:

| | | |
|---|---|---|
| $P_{transaction}$ | = | The transaction price between semiannual coupon payments |
| $C$ | = | Semiannual coupon payment amount |
| $M$ | = | Maturity value |
| $r$ | = | Semiannual required rate of return |
| $n$ | = | Total number of semiannual coupons remaining |
| $\upsilon$ | = | Days between settlement of the trade and the next coupon divided by the number of days in the coupon period. |

Note that when $\upsilon$ is equal to 1 (that is, when the next coupon payment is *exactly* six months away), the equation reduces to the standard form presented to introductory finance students:

$$P_{transaction} = \left[\sum_{t=1}^{n} \frac{C}{(1+r)^{1}(1+r)^{t-1}} + \frac{M}{(1+r)^{1}(1+r)^{n-1}}\right] = \left[\sum_{t=1}^{n} \frac{C}{(1+r)^{t}} + \frac{M}{(1+r)^{n}}\right]$$

Equation (2) consists of (a) the price of the bond immediately before the next coupon payment, and (b) a discount factor applied to the value found in part (a) over the proportion of the six-month period that the bond will be held until the next coupon payment. To illustrate, equation (2) may be rearranged to highlight the two segments:

$$(3)\quad P_{transaction} = \left[\frac{1}{(1+r)^{v}} \times \sum_{t=1}^{n} \frac{C}{(1+r)^{t-1}}\right] + \left[\frac{1}{(1+r)^{v}} \times \frac{M}{(1+r)^{n-1}}\right]$$

$$(4)\quad P_{transaction} = \frac{1}{(1+r)^{v}}\left[\sum_{t=1}^{n} \frac{C}{(1+r)^{t-1}} + \frac{M}{(1+r)^{n-1}}\right]$$

The representation of the market formula depicted in equation (4) is a primary transition point for students; and it is worth close scrutiny. The second term appears to be the normal, introductory bond price formula as of the next coupon payment date. The first term is a factor that discounts this value over the proportion of the coupon period remaining until the next coupon.

The exponents of the discount factors in the second part of equation (4) should not be overlooked. In this representation, at the time of the next coupon payment, $(t=1)$ the factor $(1+r)^{t-1}$ becomes $(1+r)^{0}$. Therefore, the next coupon payment is *not* discounted, but is *added* to the value of the bond that will exist just after the coupon payment. This amount is then discounted over the proportion of the coupon period to the day of purchase. Figure 1 displays a representation of the market process to determine the transaction price of a coupon bond between payments.

***Figure 1***
***The Transaction Price of a Coupon Bond Using the Accepted Market Formula***

| | | | | | |
|---|---|---|---|---|---|
| t=0 | | t=1 | t=2 | | n |
| --\|\|--------- ( 1 – ν ) ----------\|\|-------- ( ν ) --------\|\|-----------\|\|-------\|\|-------\|\|-------\|\|------- ......------\|\| | | | | | |
| Prior Coupon | Settlement Day | Next Coupon | | Future Coupons | Maturity |

Value just *before* next coupon: $\left[\sum_{t=1}^{n}\frac{C}{(1+r)^{t-1}}+\frac{M}{(1+r)^{n-1}}\right]$

The value is discounted using a factor that accounts for the proportion of the semiannual period that remains until the next coupon: $\frac{1}{(1+r)^{\nu}}$

***Example 1***
***Market-Determined Transaction Price of a Bond between Coupon Payments***

Assume an outstanding U.S. Treasury bond with a face value of $1,000, an annual coupon rate of 12%, and four semiannual payments remaining. Also assume a 364-day year for computational ease so that 182 days exist in each semiannual period. Further, assume investors require an annual rate of return of 8% and that it has been 72 days since the last coupon payment of $60. On the 72$^{nd}$ day after the last coupon payment, the following variable values would apply to equation (4):

| | | |
|---|---|---|
| $C$ | = | $ 60 |
| $M$ | = | $ 1,000 |
| $r$ | = | 0.04 (4% semiannual rate required) |
| $n$ | = | 4 coupon payments remaining until maturity |
| $\upsilon$ | = | The proportion of coupon period until the next coupon payment; or |

$$\upsilon = \frac{(182-72)}{182}=\frac{110}{182}=0.604395604\,.$$

With variable values applied, equation (4) is:

$$P_{transaction}=\frac{1}{(1.04)^{0.604395604}}\left[\sum_{t=1}^{4}\frac{60}{(1.04)^{t-1}}+\frac{1,000}{(1.04)^{3}}\right]$$

$$P_{transaction}=0.976573926\left[1,115.50182066\right]=\$1,089.36999$$

The ultimate solution is $1,089.37. This method will find the transaction price of a bond on any particular day between two coupon payments and is the method used by financial markets. It is somewhat awkward for students as they are not used to thinking in terms of proportions of semiannual periods (i.e. ν) from previous experience with bond valuation.

## The Quoted Price of a Bond between Coupon Payments

In an attempt to reconcile the price quoted by the financial media and the actual transaction price, some academic texts [e.g. Saunders and Cornett (2004), Francis and Ibbotson (2002)] assume that a quoted price is known. A discrete amount of daily interest is added to the given quoted price in order to arrive at the actual transaction price. Other texts [e.g. Corrado and Jordan (2005), Ross et al. (2006)], assume a transaction price is known and subtract accrued interest to arrive at the quoted price. The method used by market practitioners is not consistently clear to students.

Market convention requires that that interest on coupon bonds be accrued daily. With little variation, the texts reviewed agree with the form of Saunders and Cornett (2004). The representation of accrued interest over a semiannual period can be restated as equation (5).

$$(5)\quad AI = \sum_{k=1}^{j}\left(\frac{C}{m}\right)_k$$

Where

| | | |
|---|---|---|
| $AI$ | = | Accrued interest since the prior coupon payment. |
| $j$ | = | Number of periods (days) since the prior coupon payment. |
| $m$ | = | Number of periods (days) between promised coupon payments. |
| $C$ | = | Semiannual coupon payment dollar amount. |

The price quoted by the financial markets is determined through a combination of the transaction price of the bond on any given day between coupon payments, and the removal (subtraction) of discretely determined interest that has accrued since the prior coupon. Equations (6) and (7) are both forms of this process.

$$(6)\quad P_{quoted} = P_{transaction} - \left[\sum_{k=1}^{j}\left(\frac{C}{m}\right)_k\right]$$

$$(7)\quad P_{quoted} = \left\{\frac{1}{(1+r)^v}\left[\sum_{t=1}^{n}\frac{C}{(1+r)^{t-1}} + \frac{M}{(1+r)^{n-1}}\right]\right\} - \left[\sum_{k=1}^{j}\left(\frac{C}{m}\right)_k\right]$$

The first term in equation (7) is simply equation (4): the transaction price on any given day between two particular coupon payments. The second term indicates that a discrete amount of interest is removed from the transaction price for each day that has passed since the prior coupon payment. The result is the price quoted by the financial media.

This form is disturbing for two reasons. The first term correctly determines the transaction price of the security using investors' required rate of return as well as the compounding of semiannual interest. This is consistent with the theoretical method to determine the fair market value of the security. The second term, however, ignores compounding of interest as interest is accrued in discrete daily amounts to reduce the transaction value to the quoted price. Further, the return required by investors has no bearing on the quoted price as the return does not appear in the accrual term. This is a subtle difference between theory and practice that may make the transition difficult for students.

### *Example 2*
### *The Quoted Price of a Bond between Coupon Payments*

Financial markets follow a convention of quoting bond prices without accrued interest in order to eliminate drastic changes in the reported price on or following a coupon payment date. This practice also avoids a difficulty inherent in reporting the transaction prices of bonds that have different coupon rates, as each would accrue different amounts of interest each day between coupon payments.

Continuing the example, it has been determined that \$1,089.37 is the market-determined transaction price of a 12% coupon bond with four semiannual coupons remaining, a required annual return of 8%, when 72 days have passed since the prior coupon payment. The quoted price of this bond is found by subtracting accrued interest owed to the seller of the bond for holding it 72 days. Dividing the next $60 coupon payment by the number of days in the coupon period (182) provides the daily accrual rate of interest. Summing the result over the 72-day period provides the accrued interest that is paid by the purchaser to the seller:

$$AI = \sum_{k=1}^{j}\left(\frac{C}{m}\right)_k = \sum_{k=1}^{72}\left(\frac{60}{182}\right)_k = \sum_{k=1}^{72}\left(0.329670329\right)_k = \$\ 23.73626368$$

The quoted price of the example security starts with the transaction price on the 72nd day since the prior coupon ($1,089.37), and subtracts the interest accrued ($23.74) to arrive at a quote of $1,065.63.

## The Use of Daily Compounding of Interest to Determine the Transaction Price of a Bond between Coupon Payments

The way most finance instructors explain the pricing of a coupon bond, the fair market price can only be found if the next interest payment (coupon) is <u>*exactly*</u> one period away. In essence, students learn how to find the price of the security moments after the *current* coupon payment. It is at this point in time that the quoted price and the transaction price are equal. Instructors then tend to move on to other topics without exploring the consequences of pricing bonds that are between coupon payments.

The advent of financial calculators allows students to easily find the fair market price of a bond. Unfortunately, the nuances of the bond pricing formula may not be properly understood or appreciated by students. When framed in the proper perspective, the technology can aid in student understanding of these subtleties. It is suggested that the instructor continue the lesson about compound interest when introducing the valuation of a bond between coupon payments.

### *Example 3*
### *The Use of Daily Compounding of Interest to Determine the Transaction Price of a Bond between Coupon Payments*

The example bond has a face value of $1,000, an annual coupon rate of 12%, four semiannual payments remaining, and 182 days in each coupon payment period. If the first of four coupon payments is six months away and investors require an annual rate of return of 8%, it is a simple matter using a financial calculator to find that the fair market value of the bond is $1,072.60.

**Calculator Solution 1**
**The Price of the Example Bond after the Current Coupon Payment**

| **Calculator Key→** | **PV** | **FV** | **PMT** | **N** | **I** |
|---|---|---|---|---|---|
| **Input→** | **Solve** | 1,000 | 120/2 = 60 | 4 | 8/2 = 4 |
| **Solution→** | **1,072.5979** | | | | |
| **Notes→** | The fair price immediately after the <u>*current*</u> coupon payment. | | | | |

*Ceteris paribus*, the value of this bond will increase over the 182-day semiannual period to $1,115.50 just moments *before* the next coupon payment. This value is composed of the fair market value after the $60 coupon ($1,055.50) and the $60 coupon payment itself.

**Calculator Solution 2**
**The Price of the Example Bond after the Next Coupon Payment**

| **Calculator Key→** | **PV** | **FV** | **PMT** | **N** | **I** |
|---|---|---|---|---|---|
| **Input→** | **Solve** | 1,000 | 120/2 = 60 | 3 | 8/2 = 4 |
| **Solution→** | **1,055.5018** | | | | |
| **Notes→** | The fair price immediately after the <u>*next*</u> coupon payment. | | | | |

Assuming investors' required rate of return is held constant, the fair market value of the example bond will increase over the 182-day period from $1,072.5979 to $1,115.5018. Market practice mandates that interest on a bond between coupon payments be accrued in discrete daily amounts; therefore, the challenge is to determine the daily compounded rate of interest that will grow $1,072.5979 to $1,115.5018 over a 182-day period. Fortunately, the solution of 0.0215521567% is easily found with a financial calculator.

**Calculator Solution 3**
**Daily Compound Interest Rate between Coupon Payments**

| **Calculator Key→** | **PV** | **FV** | **PMT** | **N** | **I** |
|---|---|---|---|---|---|
| **Input→** | (1,072.5979) | 1,115.5018 | 0 | 182 | **Solve** |
| **Solution→** | | | | | **0.0215521567** |
| **Notes→** | Solution is a daily periodic rate of return. | | | | |

Knowledge of the daily periodic rate of increase in value allows the student to find the fair market transaction price of the bond on any given day *between* the two coupon payments. Assuming the bond is purchased at a price of $1,072.60 and sold 72 days later, a rational seller will expect to be compensated for the compound interest accrued over the 72-day holding period. With N = 72 and I = 0.0215521567%, the fair market price an investor should pay is $1,089.37.

**Calculator Solution 4**
**Fair Market Price between Coupon Payments**

| **Calculator Key→** | **PV** | **FV** | **PMT** | **N** | **I** |
|---|---|---|---|---|---|
| **Input→** | (1,072.5979) | **Solve** | 0 | 72 | 0.0215521567 |
| **Solution→** | | **1,089.3684** | | | |
| **Notes→** | The theoretical price of the security 72 days after it makes the prior coupon payment | | | | |

Rounding of variables aside, the result using this calculator method is *precisely* the same as the financial market method to find the transaction price of the bond. If there are no transaction costs, this value is the price that an investor will pay for the bond and that a seller will receive.

The format of this example and the use of daily compounding of interest is a natural progression for students. The topic of compound interest is usually introduced in the sequence of annual, semiannual, quarterly, monthly, daily, and perhaps continuously. While the financial market method is theoretically the same as the daily compound method introduced here, students are likely to be confused by the market method because it relies on a compound period that is a *proportion* of a semiannual period. This requires working with a semiannual term-to-maturity that is a fractional number rather than a whole number.

It is suggested that students repeat the exercise with different variables until they have convinced themselves that they have determined the transaction price a rational investor should be willing to pay for a bond between two particular coupon payments. It is then a relatively simple matter to deduct accrued interest to arrive at the quoted price that students observe in the financial media.

## Summary

It has been shown that the practitioner method used by financial markets to determine the transaction price of a coupon-paying bond is equivalent to bond pricing theory as commonly taught in introductory and upper-level finance courses. An instructor using similar examples to those presented may emphasize this theory-to-practice connection. It is suggested that students of finance may benefit from further exploration of bond pricing theory using daily compounding of interest when a bond is between coupon payments. When actual market practice is introduced, the transition from theory may be more accessible if this method is pursued. It is a transition that popular textbooks tend not to explore.

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