

Improved Pro Forma Forecasting Under Alternative Growth Rate Assumptions

Larry C. Holland¹

Abstract

Pro forma forecasting and balancing an external financing need can become inconsistent because a change in debt also changes the interest expense, which creates a circular logic problem. A second issue arises in the special case of no additional external financing at the internal growth rate (IGR), which should incorporate spontaneous financing from increases in current liabilities while maintaining a constant interest expense. Solutions are offered to both problems through a direct calculation of the interest expense without the need for an iterative process and an updated formula for the IGR, thus strengthening the literature on pro forma analysis.

Introduction

Forecasting the financial condition of a company is always a difficult process, but can be managed somewhat through a comprehensive pro forma analysis if inconsistencies are avoided. This article provides a solution to two specific problems in pro forma analysis – estimating a consistent interest expense under various growth scenarios and properly incorporating the interest expense and the self-financing from increases in current liabilities when calculating the internal growth rate.

The interest expense is a particularly difficult item to calculate correctly in estimating an external financing need as part of a pro forma analysis, especially with multiple periods. This occurs because an estimate of the interest expense is required to calculate the external financing need. Then, if a change in debt is assumed as the plug to satisfy any additional financing, the interest expense would also be expected to change proportionately with the change in debt. This seems to create a circular logic problem. A second issue occurs in the special case of no new external financing at the internal growth rate (IGR), in which the interest expense is implicitly assumed to increase with sales rather than remain constant as the level of debt remains constant. In this paper, guidance is provided for including the correct interest expense in the calculation process for both the pro forma analysis at various growth rates and for the calculation of the IGR.

This paper is divided into four sections. The first section illustrates a forecasting process using the percentage of sales method and lays the foundation for a framework to deal with inconsistencies in the calculation process. The second section shows that the pro forma interest expense can be calculated directly for any growth rate, avoiding a circular reference problem and without the need for an iterative solution. The third section illustrates that the current formula for the IGR presented in many textbooks is not consistent with a pro forma analysis in which the interest expense remains constant, and therefore understates the true IGR. A corrected formula is derived that is consistent with a constant level of debt. The last section is a summary of the results.

The Pro Forma Analysis within a Consistent Framework

A common method for developing a pro forma analysis uses the percentage of sales process. The overall process begins with a forecast for the growth in sales. Then appropriate line items on the income

¹ Professor of Finance, University of Arkansas at Little Rock, 2801 S. University, Little Rock, AR 72204, Telephone: (501)569-3042, Fax: (501) 569-8871, Email: lcholland@ualr.edu

statement and balance sheet are assumed to maintain a constant percentage of sales for the pro forma sales, which in effect means they grow at the same rate as sales. For example, it is often assumed that all items on the income statement are appropriate items to grow with sales. Also, items on the left side of the balance sheet (the assets), would also be expected to grow with sales. On the right side of the balance sheet (the liabilities and owners' equity), only the current liabilities would normally be expected to grow with sales. On a first pass calculation, the financing accounts of debt and common stock are maintained constant. The pro forma retained earnings are calculated as the previous retained earnings plus the pro forma additions to retained earnings from the income statement (i.e., net income less dividends). Then the difference between the left and right side of the pro forma balance sheet identifies any external financing need. And finally, the additional financing need is added to a plug factor (usually debt) to reach a completed financial balance in the pro forma analysis. However, a closer look at the details of the assumptions in the process reveal some of the inconsistencies and difficulties in the calculation process, specifically how interest expense and current liabilities are handled.

As stated above, it is often assumed that all items on the income statement are appropriate items to maintain a constant percentage of sales, or equivalently that they grow at the same rate as sales. However, interest expense is generally not directly related to sales. Instead, a better assumption is to relate interest expense to the level of debt.

A second issue is that it is sometimes implicitly assumed (as in some calculations of the IGR) that only the current assets and net fixed assets on the left side of the balance sheet relate to sales – this would leave out the contribution of self-financing from a change in the current liabilities on the right side of the balance sheet. Net working capital (NWC) is defined as current assets minus current liabilities. A better assumption is to relate the net working capital and net fixed assets to sales, which includes the current liabilities within net working capital.

The idea behind both of these inconsistencies falls within a broader framework of analysis which includes the concept of “invested capital” as presented by Hawawini and Viallet (2011), Firer (1999), and many others. In a traditional accounting balance sheet, total assets are defined as the sum of current assets and net fixed assets. Hawawini and Viallet (2011) on pages 19-20 of their textbook provide a different focus on a “managerial” balance sheet, which highlights invested capital rather than total assets. Invested capital can be defined as total assets minus current liabilities, or equivalently as net working capital plus net fixed assets. The basic concept is that the invested capital (which includes current liabilities) relates more closely to the operational decisions of a business. Maintaining an equality on the balance sheet, the invested capital would also be equal to the capital employed (defined as debt plus equity), which relates primarily to financing decisions.

Extending this basic framework, the return on assets (ROA) is often used as a performance measure in the traditional accounting approach, and is defined as net income divided by total assets. Firer (1999) argues that, in the context of a performance ratio such as ROA, invested capital provides “the appropriate asset base against which operating profit should be measured.” Thus, within the new framework, a parallel measure of performance is the return on capital (ROC), which is defined as net income divided by invested capital. Here, the terms capital and invested capital are used interchangeably. With the return on capital (ROC) ratio, the earnings measured as net income are related directly to a firm's total operational investments in net working capital and net fixed assets, or the invested capital. However, net income (which is in the numerator in both the ROA and ROC calculations) also includes the effect of interest expense, which relates to the level of debt within employed capital. It is important to separate the effect of capital structure (i.e., the amount of debt) from the earnings, which are normally defined by net income. The earnings before interest and taxes (EBIT) is defined as sales minus the total cost of those sales. The total cost of sales includes cost of goods sold (CGS), selling, general and administrative expense (SG&A), and depreciation. Including the effect of taxes but not interest expense gives a measure of earnings that is independent of the capital structure, EBIT (1-t), which is also known as the net operating profit after taxes (or NOPAT). This finally leads to the performance measure of return on invested capital (ROIC). ROIC is defined as NOPAT divided by invested capital, or $EBIT(1-t)/IC$. ROIC is thus free from an influence from interest expense or debt in the numerator, and also includes the self-financing feature of current liabilities in the denominator.

In summary, the concepts of invested capital (IC), return on capital (ROC), and return on invested capital (ROIC) provide a more consistent framework for calculations within a pro forma analysis, compared to a traditional accounting approach based on total assets and return on assets (ROA). This more consistent framework will become an integral part of a solution to an inconsistent interest expense as part of pro forma analysis and of an internal growth rate formula that implicitly includes the self-financing of current liabilities. Both of these issues are explored in more detail in the next two sections.

Calculating the Pro Forma Interest Expense for any Growth Rate

As mentioned in the previous section, a simplistic pro forma analysis would relate the interest expense to the level of sales, which is often inconsistent with the level of debt. A better approach is to relate interest expense to the level of debt. However, the correct level of interest expense is difficult to calculate if the level of pro forma debt does not remain constant. For example, this difficulty occurs frequently if debt is used as the plug factor to reach a financial balance. At first it seems that the interest expense is necessary to determine the external financing need and the subsequent level of new debt. Yet the final level of new debt is necessary to determine the interest expense, which seems to introduce a circular argument. This difficulty is also stated in various textbooks. For example, Cornet, Adair, Nofsinger (2012) p. 511, 514 state that interest expense and additional funding needed involve a circular reference and require an iterative calculation, which they describe using Excel. Brigham and Ehrhardt (2005) p. 485 state that interest expense causes circularity in a spreadsheet, which results in financing feedback. They suggest that one solution to avoid this circularity is to use the beginning of period debt, although this would not be truly consistent and could underestimate the interest expense for a growing company.

However, the correct interest expense can be directly determined on the first pass calculation without the need for an iteration. This is possible because there are two equations relating the interest expense and the final level of debt. The first equation is the definition of interest expense as the interest rate times the level of debt. The second equation is the accounting equality that requires the increase in liabilities and owner's equity to equal the increase in assets. The interest expense enters through the additions to retained earnings. With two equations and two unknowns, the interest expense can be solved directly. To solve for the interest expense, one can either specify the pro forma EBIT to sales margin, sales to invested capital ratio, tax rate, and interest rate, or more simply assume steady state conditions in which these variables are constant. Under steady state conditions, Appendix 1 shows the derivation of the correct interest expense for any level of growth as

$$INT_{t+1} = \frac{i [Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1}]}{1 - i(1 - t) b_{t+1}} \quad (1)$$

- where
- INT_{t+1} = the pro forma interest expense for year t+1,
 - i = the interest rate on debt in the actual year t,
 - $Debt_t$ = the debt for the actual year t,
 - g = the growth rate in sales,
 - IC_t = the invested capital in the actual year t,
 - $EBIT_t$ = the earnings before interest and taxes in the actual year t,
 - b_{t+1} = the retention rate for year t+1, and
 - t = the tax rate.

Equation 1 can be applied in a straightforward manner within a pro forma analysis. To demonstrate the value of this Equation 1, Table 1 shows an illustration of developing a pro forma forecast from financial statements for an example company at two arbitrary growth rates, 5% and 15%. The first column shows the income statement and balance sheet for the example company for an actual year. The forecast in Pro Forma 1 uses a 5% growth rate and follows the description given earlier in which the

interest expense is incorrectly assumed to grow with sales, but allows the current liabilities to grow with sales. Pro Forma 1 shows a surplus (i.e., negative financing need) of 4.45, which is used to reduce the level of debt from 50 to 45.55. However, with the assumption that interest expense grows with sales, the interest expense is shown to increase to 5.25 despite the decrease in the level of debt. The implied interest rate can be calculated by dividing the interest expense by the level of debt. In the actual data, the implied interest expense is $5/50 = 10\%$. However, the implied interest rate in Pro Forma 1 is shown to inconsistently increase to $5.25/45.44 = 11.5\%$, breaking from a relationship with the level of debt.

In Pro Forma 2 and Pro Forma 3, the pro forma interest expense is calculated on the first pass using Equation 1. Any external financing need is again assumed to be satisfied through a change in debt. In the 5% growth case for Pro Forma 2, the forecast shows a surplus (i.e., a negative financing need) of 4.71, which is again used to reduce the level of debt – in this case from 50 to 45.29. Note, however, the implied interest rate of $4.53/45.29 = 10\%$ does not change from the actual implied interest rate of 10%. Thus, the interest expense more appropriately remains related to the level of debt rather than to the growth in sales. Pro Forma 3 illustrates that the use of Equation 1 at other growth rates continues to maintain the relationship of interest to the level of debt. Pro Forma 3 applies a forecast of 15% growth in sales. This forecast shows an external funding need of 4.54, which is added to the level of debt from 50 to 54.54 to support the extra growth. Again, the implied interest rate is $5.45/54.54 = 10\%$. Note in the completed balance, the pro forma interest expense remains at 10% of the level of debt in both Pro Forma 2 and Pro Forma 3. This demonstrates that the formula works appropriately without the need of an iterative solution.

One of the added benefits of a direct solution for the interest expense is that it can simplify a pro forma analysis, especially if multiple future periods are involved. For example, a valuation analysis will often include a forecast for a number of years in the future, and it is necessary to maintain a consistent accounting relationship in the data in order to achieve a consistent final result. Therefore, a direct equation for the interest expense can eliminate a circular reference and the need for an iterative solution, which simplifies the analysis.

The Special Case of the IGR

The IGR is defined as the growth rate in which a firm reaches a financial balance without the use of any external funding. Specifically, this means using only internally generated funds without any issuance of new debt (i.e., constant existing debt) and without any issuance of new equity or repurchases. Therefore, the IGR is a special case in which both the existing debt and the interest expense would remain constant. The simplified formula for calculating the IGR most often published in textbooks is

$$IGR = \frac{b ROA}{1 - b ROA} \quad (2)$$

- where
- IGR = the internal growth rate,
 - b = the retention ratio, or the fraction of net income retained in the firm,
 - ROA = the return on assets, defined as the NI/Assets,
 - NI = net income, and
 - Assets = the total assets of a firm.

However, this formula for the IGR is implicitly based on weak or inconsistent assumptions, as will shortly be demonstrated. Yet the formula is often presented without much explanation or discussion of the shortcomings. For example, Ross, Westerfield, Jordan (2013) p. 107 and Cornet, Adair, Nofsinger (2012) p. 90 show the same formula for the IGR as in Equation 2. Berk, DeMarzo, Harford (2012) p. 548 shows that the $IGR = b (ROA)$ and Brealy, Myers, Marcus (2012) p. 517 shows that the $IGR = b (ROE)$ (Equity/Assets), both of which point out in a footnote that the ROA or the ROE must be calculated using the beginning of period Assets or Equity (i.e., data from last year's balance sheet).

This is consistent with the clarification in Angell (2011) that the calculation of the IGR (and the sustainable growth rate, or SGR) must use the form in Equation 2 if the latest balance sheet data is used.

A simplified formula for calculating the IGR can be very useful. However, it does not always reveal the implicit assumptions incorporated within the simplified formula. Fortunately, a pro forma analysis can clearly identify the implicit assumptions through a calculation process similar to the pro forma analysis in the previous section. An assumption of a steady state condition is normally made in the calculation of the IGR, with a constant retention ratio, EBIT to sales margin, sales to asset ratio, tax rate, and interest rate (or more specifically, the interest expense to debt ratio). Table 2, Panel A, shows the calculation of the IGR applying the formula in Equation 2 (labeled as IGR_1) and using the actual financial statements from the example in Table 1. Then Table 3 shows a pro forma analysis using the calculated IGR (8.108% in this example) as the growth rate under steady state conditions.

The Pro Forma 4 example in Table 3 shows that the formula for the IGR in Equation 2 will reach a financial balance on the first pass calculation, but only if it is implicitly assumed that interest expense grows with sales rather than debt and that current liabilities remain constant in order to reach a financial balance with no change in the level of debt. Once again the implied interest rate changes from 10% in the actual year to 10.8% if Equation 1 is not used to calculate the interest expense. A better set of assumptions would maintain the relationship of interest expense to the level of debt and incorporate an increase in the current liabilities such that the net working capital grows with sales (or equivalently that the net working capital to sales ratio remains constant as sales grow). Equation 2 does not include the effect of the self-financing of current liabilities because only the total assets are in the denominator of the ROA calculation. A more inclusive approach is to include the return on capital (ROC), where capital is defined as total assets less current liabilities (or equivalently the net working capital plus fixed assets). Equation 2 also implicitly assumes that the interest expense will increase proportionately to a sales increase. In reality, the definition of the IGR states that debt remains constant. Therefore, one would expect that the interest expense would remain constant as well, assuming a constant interest rate on the debt (or more precisely, the interest expense to debt ratio).

The Pro Forma 5 example in Table 3 changes the key assumptions by calculating the interest expense through Equation 1 at the same calculated growth rate (IGR_1), and allowing the current liabilities as well as the current assets to increase with sales. This example shows that the formula in Equation 2 for calculating the IGR no longer reaches a balance in the first pass calculation and underestimates the true IGR. The completed balance indicates a surplus of funds such that the level of debt would decrease at that growth rate, which is inconsistent with the definition of the IGR.

Appendix 2 derives a new formula for the IGR that includes both the self-financing feature from an increase in current liabilities and the assumption that interest expense remains constant. This new formula is

$$IGR = \frac{b \text{ ROC}}{1 - b \text{ ROIC}} \quad (3)$$

where

- ROC = the return on capital, defined as $ROC = NI/IC$,
- IC = Invested Capital, defined as the total assets of the firm minus the current liabilities (equivalently, net working capital plus net fixed assets).
- ROIC = the return on invested capital, defined as $ROIC = EBIT(1-t)/IC$,
- EBIT = the earnings before interest and taxes, and
- t = the tax rate.

Table 2, Panel B, uses the new formula in Equation 3 to calculate the IGR (labeled as IGR_2), which is somewhat higher than the previous calculation in the example. The Pro Forma 6 example in Table 3 shows that an appropriate balance is reached on the first pass calculation when the interest expense is calculated using Equation 1 and current liabilities are allowed to grow with sales. Note that the interest

expense calculated by Equation 1 remains constant, and the level of debt also remains the same because a zero level of external funding needs is calculated on the first pass. Thus, this example shows that the new formula in Equation 3 implicitly incorporates more appropriate assumptions for the interest expense and current liabilities. Equation 3 therefore provides a superior calculation of the IGR, free of the weakness and inconsistency of the existing formula for the IGR.

Summary

The current state of pro forma analysis suffers from two problems – inappropriate assumptions concerning the interest expense due to a circular logic problem and the failure to incorporate spontaneous funding from increases in current liabilities. Solutions to these two problems are provided. In the first case, a formula is derived which directly calculates the pro forma interest expense. This eliminates the complexity of a circular reference and removes the need for an iterative solution or other approximation of the pro forma interest expense. Thus, a direct formula for the interest expense simplifies the calculations of a pro forma analysis, which is particularly useful when multiple future periods are involved.

The second issue is that the formula for calculating the IGR as presented in many textbooks does not incorporate the self-financing from an increase in current liabilities and is not consistent with an assumption of constant debt (and therefore a constant interest expense). A corrected formula for the IGR is derived which yields a financial balance of zero external financing need when sales grow at the internal growth rate and the interest expense and debt remain constant. Both of these new formulas provide tools for a more straightforward pro forma process, thus strengthening the literature on pro forma analysis for effective financial forecasting and planning.

References

- Angell, Robert J. 2011. "A Note on the Calculation of Sustainable Growth Rates in Finance Texts." *Journal of Economics and Finance Education* Vol. 10, Number 1, Summer: 41-44.
- Berk, Johathan, Peter DeMarzo, and Jarrad Harford. 2012. *Fundamentals of Corporate Finance* (2nd Ed.). Pearson Education, Inc., Boston, MA
- Brealey, Richard A., Stewart C. Myers, and Alan J. Marcus (2012). *Fundamentals of Corporate Finance* (7th Ed.). McGraw-Hill Irwin, New York, NY.
- Brigham, Eugene F. and Michael C. Ehrhardt. 2005. *Financial Management: Theory and Practice* (11th ed.), Thomson Southwestern, Mason, OH.
- Cornett, Marcia Millon, Troy A. Adair, Jr., and John Nofsinger (2012), *Finance Applications & Theory* (2nd Ed.). McGraw-Hill Irwin, New York, NY.
- Firer, Colin, (1999), "Driving Financial Performance Through the DuPont Identity: A Strategic Use of Financial Analysis and Planning," *Financial Practice and Education*, Vol 9, Issue1, Spring/Summer, p. 34-45.
- Hawawini, Gabriel and Claude Viallet. 2011. *Finance for Executives: Managing for Value Creation* (4th Ed.). South-western Cengage Learning, Mason, OH.
- Ross, Stephen A., Randolph W. Westerfield, and Bradford D. Jordan (2013), *Fundamentals of Corporate Finance* (10th ed.). McGraw-Hill Irwin, New York, NY.

Table 1: Pro Forma Financial Statements for Two Arbitrary Growth Rates

Key Assumptions

Interest Expense	Grows with Sales	Calculated Eq. 1	Calculated Eq. 1
Current Liabilities	Grow with Sales	Grow with Sales	Grow with Sales

		Pro Forma 1	Pro Forma 2	Pro Forma 3
	Actual	g = 5%	g = 5%	g = 15%
Sales	100	105.00	105.00	115.00
CGS	40	42.00	42.00	46.00
SG&A	20	21.00	21.00	23.00
Depreciation	10	10.50	10.50	11.50
<u>EBIT</u>	<u>30</u>	<u>31.50</u>	<u>31.50</u>	<u>34.50</u>
Interest Expense	5	5.25	4.53	5.45
<u>EBT</u>	<u>25</u>	<u>26.25</u>	<u>26.97</u>	<u>29.05</u>
Taxes @ 40%	10	10.50	10.79	11.62
<u>Net Income</u>	<u>15</u>	<u>15.75</u>	<u>16.18</u>	<u>17.43</u>
x b	x 0.6	x 0.6	x 0.6	x 0.6
<u>Additions to RE</u>	<u>9</u>	<u>9.45</u>	<u>9.71</u>	<u>10.46</u>
Current Assets	60	63.00	63.00	69.00
Net Fixed Assets	60	63.00	63.00	69.00
<u>Assets</u>	<u>120</u>	<u>126.00</u>	<u>126.00</u>	<u>138.00</u>
First Pass				
Current Liabilities	20	21.00	21.00	23.00
Debt	50	50	50	50
Common Stock	10	10	10	10
Retained Earnings	40	49.45	49.71	50.46
<u>Liabilities & OE</u>	<u>120</u>	<u>130.45</u>	<u>130.71</u>	<u>133.46</u>
External Financing Need		-4.45	-4.71	4.54
Completed Balance				
Current Liabilities	20	21.00	21.00	23.00
Debt	50	45.55	45.29	54.54
Common Stock	10	10	10	10
Retained Earnings	40	49.45	49.94	50.46
<u>Liabilities & OE</u>	<u>120</u>	<u>126.00</u>	<u>126.00</u>	<u>138.00</u>

$$\text{Interest Rate} = \frac{INT}{Debt}$$

$$\frac{5.45}{54.54} = 10\% \quad \frac{5}{50} = 10\% \quad \frac{5.25}{45.55} = 11.5\% \quad \frac{4.53}{45.29} = 10\%$$

Table 2: Two Calculations of the IGR

Panel A: Uncorrected Calculation of IGR

$$IGR_1 = \frac{b ROA}{1 - b ROA} = \frac{.6 (.125)}{1 - .6(.125)} = 8.108\%$$

where the return on assets is $ROA = \frac{NI}{Assets} = \frac{15}{120} = 12.5\%$
 and the retention ratio is $b = 0.6$

Panel B: Corrected Calculation of IGR

$$IGR_2 = \frac{b ROC}{1 - b ROIC} = \frac{.6 (.15)}{1 - .6(.18)} = 10.090\%$$

where the return on capital is $ROC = \frac{NI}{IC} = \frac{15}{100} = 15\%$
 and the return on invested capital is $ROIC = \frac{EBIT(1-t)}{IC} = \frac{30(1-.4)}{100} = 18\%$

Table 3: Pro Forma Financial Statements for Two Different IGR Calculations

Key Assumptions		Grows with Sales Constant	Calculated Eq. 1 Grow with Sales	Calculated Eq. 1 Grow with Sales
Interest Expense Current Liabilities		Pro Forma 4 IGR ₁ = 8.108%	Pro Forma 5 IGR ₁ = 8.108%	Pro Forma 6 IGR ₂ = 10.090%
	Actual			
Sales	100	108.11	108.11	110.09
CGS	40	43.24	43.24	44.04
SG&A	20	21.62	21.62	22.02
Depreciation	10	10.81	10.81	11.01
<u>EBIT</u>	<u>30</u>	<u>32.43</u>	<u>32.43</u>	<u>33.03</u>
Interest Expense	5	5.41	4.82	5.00
<u>EBT</u>	<u>25</u>	<u>27.03</u>	<u>27.62</u>	<u>28.03</u>
Taxes @ 40%	10	10.81	11.05	11.21
<u>Net Income</u>	<u>15</u>	<u>16.22</u>	<u>16.57</u>	<u>16.82</u>
x b	x 0.6	x 0.6	x 0.6	x 0.6
<u>Additions to RE</u>	<u>9</u>	<u>9.73</u>	<u>9.94</u>	<u>10.09</u>
Current Assets	60	64.86	64.86	66.05
Net Fixed Assets	60	64.86	64.86	66.05
<u>Assets</u>	<u>120</u>	<u>129.73</u>	<u>129.73</u>	<u>132.11</u>
First Pass				
Current Liabilities	20	20	21.62	22.02
Debt	50	50	50	50
Common Stock	10	10	10	10
Retained Earnings	40	49.73	49.94	40.09
<u>Liabilities & OE</u>	<u>120</u>	<u>129.73</u>	<u>131.56</u>	<u>132.11</u>
External Financing Need		0.0	-1.83	0.0
Completed Balance				
Current Liabilities	20	20	21.62	22.02
Debt	50	50	48.17	50
Common Stock	10	10	10	10
Retained Earnings	40	49.73	49.94	50.09
<u>Liabilities & OE</u>	<u>120</u>	<u>129.73</u>	<u>129.73</u>	<u>132.11</u>
Interest Rate = $\frac{INT}{Debt}$	$\frac{5}{50} = 10\%$	$\frac{5.41}{50} = 10.8\%$	$\frac{4.83}{48.17} = 10\%$	
	$\frac{5}{50} = 10\%$			

Appendix 1: Derive the Pro Forma Interest Expense for any Growth Rate

Let g be the pro forma growth in sales. Assume a constant ratio of Sales/IC, where invested capital (IC) is defined as total assets less current liabilities (or equivalently the sum of debt plus equity). Then the growth rate for invested capital is the same as the growth rate in sales, or

$$IC_{t+1} = (1 + g) IC_t \quad (4)$$

Rearranging the terms shows that

$$g (IC_t) = IC_{t+1} - IC_t \quad (5)$$

Also assume no new issuance of equity or repurchases and clean surplus accounting, where all changes in retained earnings (RE) pass through the income statement (i.e., $\Delta RE = NI - DIV$). Then the growth in equity is solely from the addition to retained earnings (ARE) from the income statement. Because the invested capital also equals the sum of debt plus equity, the pro forma increase in invested capital must also equal the pro forma increase in debt plus the addition to retained earnings, or

$$g IC_t = Debt_{t+1} - Debt_t + ARE_{t+1} \quad (6)$$

Rearranging the terms yields

$$Debt_{t+1} = Debt_t + g IC_t - ARE_{t+1} \quad (7)$$

Assuming a constant tax rate (t), the pro forma addition to retained earnings is defined as

$$ARE_{t+1} = (EBIT_{t+1} - INT_{t+1})(1 - t) b_{t+1} \quad (8)$$

Assuming a constant EBIT margin (EBIT/Sales) means that EBIT would grow at the same growth rate as sales, or $EBIT_{t+1} = (1+g) EBIT_t$. Substituting and rearranging the terms yields

$$ARE_{t+1} = (1 + g) EBIT_t(1 - t) b_{t+1} - INT_{t+1}(1 - t) b_{t+1} \quad (9)$$

Substituting Equation 9 into Equation 7 yields

$$Debt_{t+1} = Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1} + INT_{t+1}(1 - t) b_{t+1} \quad (10)$$

Assuming a constant interest rate ($i = \text{Interest Expense}/\text{Debt}$),

$$INT_{t+1} = i Debt_{t+1} \quad (11)$$

Equations 10 and 11 represent two equations with two unknowns (INT_{t+1} and $Debt_{t+1}$). To solve these two equations for INT_{t+1} , substitute $Debt_{t+1}$ from Equation 10 into Equation 11, which yields

$$INT_{t+1} = i [Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1} + INT_{t+1}(1 - t) b_{t+1}] \quad (12)$$

Separating the INT_{t+1} term on the right side yields

$$INT_{t+1} = i [Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1}] + i [INT_{t+1}(1 - t) b_{t+1}] \quad (13)$$

Subtracting $i [INT_{t+1}(1 - t) b_{t+1}]$ from both sides yields

$$INT_{t+1} - i [INT_{t+1}(1 - t) b_{t+1}] = i [Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1}] \quad (14)$$

Combining the terms with INT_{t+1} and then solving for INT_{t+1} yields

$$INT_{t+1} = \frac{i [Debt_t + g IC_t - (1 + g) EBIT_t(1 - t) b_{t+1}]}{1 - i(1 - t) b_{t+1}} \quad (15)$$

Appendix 2: Derive the IGR

The definition of the IGR is the growth in sales that yields a financial balance of zero external financing need, i.e. no issuance of new debt or new equity. Invested capital (IC) can be defined as total assets less current liabilities (or net working capital plus net fixed assets). Then if the Sales/IC ratio remains constant, invested capital will grow at the same rate as sales, or

$$IC_{t+1} = (1 + IGR) IC_t \quad (16)$$

Rearranging the terms shows that

$$IGR (IC_t) = IC_{t+1} - IC_t \quad (17)$$

By definition, with no issuance of new debt or new equity, the growth in invested capital must be solely from the additions to retained earnings. Net income is defined as $NI = (EBIT-INT)(1-t)$, and the additions to retained earnings is defined as $b(NI)$, where b is the retention ratio. If the tax rate and retention rate remain constant, then

$$IGR (IC_t) = [EBIT_{t+1} - INT_{t+1}] (1 - t) b \quad (18)$$

With constant debt, the interest expense will remain constant. Also, if the EBIT margin (EBIT/Sales) remains constant, then the growth rate in EBIT will be the same as the growth rate in sales. Substituting $INT_{t+1} = INT_t$ and $EBIT_{t+1} = (1+IGR) EBIT_t$ yields

$$IGR (IC_t) = [(1 + IGR) EBIT_t - INT_t] (1 - t) b \quad (19)$$

Rearranging the terms on the right side yields

$$IGR (IC_t) = (EBIT_t - INT_t) (1 - t) b + (IGR) EBIT_t (1 - t) b \quad (20)$$

Substituting $NI = (EBIT-INT)(1-t)$ and dividing by IC yields

$$IGR = b \frac{NI_t}{IC_t} + IGR b \frac{EBIT_t(1-t)}{IC_t} \quad (21)$$

Define the return on capital (ROC) as $ROC_t = NI_t/IC_t$ and the return on invested capital (ROIC) as $ROIC_t = EBIT_t (1-t)/IC_t$. Substituting into the Equation 21 yields

$$IGR = b ROC_t + IGR (b ROIC_t) \quad (22)$$

Solving for IGR yields

$$IGR = \frac{b ROC_t}{1 - b ROIC_t} \quad (23)$$