<1> Econ Beats: A Semester Long, Interdisciplinary, Project-Based Learning Assignment  
   Abdullah Al-Bahrani, Bradley Libis, Sara Drabik, and John Gibson

<11> The Relative Value of AER P&P Economic Education Papers  
   J.R. Clark, Joshua C. Hall, and Ashley S. Harrison

<20> Teaching the Paradox of Thrift with a Water Rescue Analogy  
   Ben L. Kyer and Gary E. Maggs

<23> A Class Activity on the Income and Substitution Effects of a Price Change  
   Philip Sirianni

<35> Highly Visual, Concept-Mapped Presentations for Microeconomics with Prezi  
   Thomas P. Andrews and Kristen Doran

<44> Cryptocurrencies in a Money and Banking Course  
   Jennifer W. Kelber and Meghan Hennessy Mihal

<56> The Benefits of Recognizing Interest Rates Independent of Goods Market Equilibrium  
   S. Kirk Elwood

<68> Credit Default Swap –Pricing Theory, Real Data Analysis and Classroom Applications Using Bloomberg Terminal  
   Yuan Wen and Jacob Kinsella

<79> Designing Effective Examples of Optimal Capital Structure Estimation for the Intermediate Corporate Finance Course  
   Robert B. Burney

<88> Obtaining Consistent Corporate Valuation  
   William A. Ogden, Pedro Sottile, and Srinivasan Sundaram

Academy of Economics and Finance
Econ Beats: A Semester Long, Interdisciplinary, Project-Based Learning Assignment

Abdullah Al-Bahrani, Bradley Libis, Sara Drabik, John Gibson

Abstract

Innovative teaching in economic education has used media, music, and popular culture. We provide educators with a semester-long, interdisciplinary, collaborative, active learning project called Econ Beats. The project requires students from Economics to work with media students over an entire semester to create a music video that explains economic content. The highlight of project is a public screening that showcases students’ projects. We provide a blueprint to educators on how to best incorporate the assignment and plan the event. The project brings attention to economics, and is a way to engage the community.

Introduction

Today’s employers expect that students graduating with a college degree are equipped with the skills to tackle complex real-world problems (Steiner and Laws 2006). The education system had previously attempted to prepare students by creating discipline-specialized graduates (Remington-Doucette 2013). While specialized students are better equipped for success in graduate school, they might not be as prepared to work in areas that require exposure to varied skills and knowledge. A more comprehensive education system allows students to be exposed to a variety of skills, and allows them to gain a more rounded understanding of a subject. Some universities have shifted to interdisciplinary learning and emphasized the importance by requiring degrees, programs, curriculum, or assignments that integrate two or more departments on campus. This interdisciplinary method of teaching allows students to apply their understanding in different dimensions and allows students to gain a deeper level of understanding (Lim et al. 2012). Other institutions have utilized project-based learning to enhance students understanding of real-world problems. Project-based learning allows students to be active learners and requires a higher level of engagement with the material (Kleinberg 2008; Salemi 2002; Beaudin et al 2017).

While the benefits of interdisciplinary learning and project-based teaching may be attractive to educators, the learning curve is steep, it requires creative implementation and the ability to identify areas on campus that can work together. Business education has previously achieved little progress in incorporating interdisciplinary teaching due to the efforts required to change the curriculum and reform the culture of education in Business Colleges (Bajada and Trayler 2013; Fish 2013).

In this paper we introduce Econ Beats, an interdisciplinary project that merges economics, music, media, and communication. This semester-long project can be incorporated within class assignments. Educators interested in interdisciplinary teaching or project-based assignments can introduce it in their
classes at a relatively lower cost than more comprehensive curriculum change. We provide a blueprint on how we incorporated the assignment, how we assess it, why we believe the assignment is beneficial, and some recommendation and challenges based on our experience. We provide insight based on two years of experience and interaction with students enrolled in 9 sections of 2 classes. Over the two years roughly 300 students were involved with the project. We expect that at most universities, media, communication, music and economics rarely interact, and the Econ Beats assignment will allow for faculty and students to benefit from a more comprehensive and engaging approach to learning.

Why Incorporate Econ Beats?

Econ Beats is a great opportunity to introduce interdisciplinary teaching, project-based assignments, and to reinforce the importance of working in groups. At most institutions, Principles of Economics courses are either at the freshman or sophomore level, and often fulfill general education requirements. While many students are enrolled in economics courses across the nation, few go on to graduate with a major in economics (Allgood, Walstad and Siegfried 2015). Therefore, it is important that the students that are exposed to economics are able to retain the most important concepts and be able to communicate the ideas effectively. Studies have suggested that using the Arts and Humanities can help increase student retention, and student understanding of economics (Cotti and Johnson 2012; Hartley 2001; Wattsee 2002) Al-Bahrani et al. (2016) provide a list of low cost Art assignments to incorporate in the economics classroom. Relative to Al-Bahrani et al. (2016) Econ Beats is a more involved project. It requires a higher level of coordination and it meets the semester long, interdisciplinary, and project based requirements for an effective classroom assignment.

Econ Beats is a semester-long project that requires students to incorporate information as they learn it. Valcarcel (2013) and Beaudin et al. (2016), suggest that a semester long project is better at preparing students for jobs relative to small projects spread out over the semester. Furthermore, Econ Beats leverages the power of competition. It is suggested that competition helps increase students’ job preparedness (Aguilar and Soques 2015; Gulley and Jackson 2015; Singh and Russo 2013; Beaudin et al. 2016). We believe the structure of the assignment can help increase interest in economics and its application.

While the structure of Econ Beats may increase students’ job preparedness, we expect that students participating in Econ Beats would experience an improvement in their performance and learning outcomes for the courses. We do not formally test for learning improvements in this paper, however researchers suggest for the current generation, the digital natives, learning is enhanced in environments where teaching is relevant, dynamic, and utilizes active learning (Gros and Lopez 2016). Experimental analysis of the efficacy of new learning strategies is difficult (Cottie and Jonshon 2012). Often the first stage of innovation is to incorporate it into teaching. Assessing how effective the teaching methods are usually follows after its implementation. Table 1 provides a list of studies that use the Arts or Humanities to teach economics. When available, we provide the impact of the teaching method on learning outcomes. The table indicates two things. First, research introducing innovative teaching methods is descriptive and rarely includes evidence of learning outcomes. Secondly, when assessed, teaching methods using the Arts and Humanities in teaching increases students’ outcomes, as measured by learning and interest.
Table 1: Summary of select papers that examine the efficacy of the use of Arts and Humanities to teach economics.

<table>
<thead>
<tr>
<th>Author</th>
<th>Learning Tool</th>
<th>Data</th>
<th>Findings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Al-Bahrani, Dowell, Patel (2016)</td>
<td>Scrapbooking/video</td>
<td>Non-empirical</td>
<td>Provide several different video scrapbook assignment options that allow for instructors to integrate into the economics curriculum.</td>
</tr>
<tr>
<td>Cotti and Johnson (2012)</td>
<td>Historical Novels</td>
<td>Sample Size of 99 students</td>
<td>Description of how to incorporate a historical novel into a microeconomics class and provide some evaluation strategies. They find no difference in learning but difference in student interest in economics increases when exposed to literature.</td>
</tr>
<tr>
<td>Hall and Lawson (2008)</td>
<td>Music</td>
<td>Non-empirical</td>
<td>Describes a writing assignment designed to leverage student interest in music to improve economic understanding.</td>
</tr>
<tr>
<td>Harter (2003)</td>
<td>Music/Writing</td>
<td>Sample size of 77 students.</td>
<td>There is a relationship between learning style and learning outcomes. Music can help improve some students learning.</td>
</tr>
<tr>
<td>McClough and Heinfeldt (2012)</td>
<td>Music</td>
<td>Sample size 43 students.</td>
<td>Conduct a study to examine the effectiveness of using music to convey economic concepts. The empirical results suggest using music lyrics improve student understanding</td>
</tr>
</tbody>
</table>
We hypothesize that students would benefit from Econ Beats. We expect it to increase student learning because it 1) is interdisciplinary 2) is a semester long project 3) relies on group work 4) leverages music 5) leverages competition. 6) requires students to be active participants in the learning process.

The Assignment

Econ Beats was assigned at Northern Kentucky University during the academic years 2015-2017. The assignment is an extension of a project introduced by Holder et al. (2015). The original assignment requires students to remake a song and rewrite the lyrics by incorporating economic concepts. Through Econ Beats we extend the project by matching economics students with students from Media and Broadcasting (EMB) class. The EMB students are required to develop a vision, shoot the video, and record the audio. For EMB students the learning objective is to learn how to work with a real world client. They are exposed to project management, working with clients that are not exposed to technical aspect of media and production, and team management aspects.

There are several learning objectives for the economics students. First, it is expected that they learn and understand economic terminology. Second, they will learn how to communicate economic concepts to the general public. They will learn how to work with groups and, finally, how to use technology. The assignment is designed to help students develop their communication and presentation skills. Traditionally, these learning objectives are met through requiring students to prepare presentation. We believe that Econ Beats provides an alternative approach for instructors that are looking to extend their assignments beyond traditional methods.

The assignment was administered at the principle level courses in economics. The EMB students were enrolled in Single Camera Production, a class that focuses on creating audio and video content. The assignment requires a high level of coordination within classes and across subjects. Students are provided with a rubric, and a list of assignments to help them arrive to the end project. The culmination of the project is celebrated with a public screening of all submissions. Great effort was devoted to ensure that the projects have the same weight in both classes. The project accounts for 20% of the overall grade in both, economics and EMB.

The number of projects vary by semester. In Fall 2015 there were 8 completed projects, 7 in the Spring of 2016, and 8 in the Fall of 2016. Groups consist of 7-8 economics students, and are matched with 4 students from EMB. At Northern Kentucky University, Principle of Economics courses sizes are between

---

2 Economics instructors can collaborate with other departments. Music, Theater, English, and Communications Department can serve as possible partners.
50-55 students, while EMB classes are restricted to 20-25 students. The group sizes are larger than optimal, but we are constrained by the number of EMB students.

Students are provided with the rubric (see appendix), and examples of previous assignments on the first day of class. They are given 14 weeks and a timeline to complete the assignment and can use any content discussed in class, book, or supplemental reading. The lyrics have to be sufficiently remade and include a “heavy dose of economics”. Economics students must carefully communicate their content so that the EMB students can create a video based on the lyrics and concepts. This part of the project is the most important as economics students must learn how to take difficult concepts and teach it to students that have never been formally exposed to economics previously. This process helps reinforce the concepts for economics students. For EMB students, this is usually their only exposure to economics during their undergraduate career.

The literature on economic education has highlighted the importance of active learning in increasing student retention of the subject (Hoyt 2003). We believe Econ Beats has the potential to help students gain economic knowledge due to the active learning format of the project, and Econ Beats also makes economics more accessible to the general public. Given that the current generation struggles to find the relevancy of economics, Econ Beats can increase awareness and attract new students to the principles classes. While learning outcomes are expected to increase with the use of Econ Beats, we have not formally assessed the efficacy of the assignment. Future research in this area should formally examine the returns to using this assignment.

Timeline of Assignments and Challenges

Through several iterations of the project, we have found that there are some rules that help better coordinate teams within class and across courses. Our first suggestion is to randomly select group members. We randomly select students in the economics class into groups, then each group is randomly matched with a group from EMB. We have found that by allowing members to self-select the dynamic of the group suffers. When self-selecting, students will often create groups with their friends. However, rarely will a group be comprised of friends only. Students that can not find a group are added to already established groups. This creates a situation where the added student never fully integrates. Instructors often avoid group projects due to the complexity of assigning groups (Behesthtian-Ardekani and Mahmood 1986). Although studies have shown that there are more effective ways to assign groups (Muller 1989), we find random selection to be efficient for this project.

Since groups are randomly selected, we advise that the instructor facilitates the introduction. To encourage economics students to meet, we task each group with selecting their favorite Econ Beats video from previous semesters. Each group is required to submit a one-page summary that details which group members were in attendance, which video they picked, and why they chose the videos. The structure of this assignment requires them to meet, it also exposes them to previous submission which serve as a benchmark, and requires them to identify a person to submit the assignment on behalf of the group. The following week we facilitate and organize the EMB and economics group meeting. We recommend making the introduction between the EMB and economics groups as early as possible. In our timeline, we have devoted the third week of classes as the week which economics and EMB students meet.

At the seventh week of the semester we require students to submit the instrumental audio of the song that they have selected. This serves as a commitment strategy and a reminder that they are expected to be working on the project. Smaller low-stake task help keep the project at the forefront of students’ busy schedule. Through our experience we have learned that is students are left to their own planning they will procrastinate. Waiting to complete the Econ Beats project in a short time reduces the quality of the project. Econ Beats is too complex and requires students to collaborate to successfully complete the project. Our objective is to teach students how to meet deadlines and how to manage projects.

Groups are required to submit their lyrics by the ninth week of class. Students are encouraged to seek help to ensure that economic terms are used correctly and that they have enough economic content to receive credit for the assignment. The audio file with the vocals is due shortly after. This helps the instructor provide feedback on the quality of the audio and whether the students need to rerecord it. This responsibility can be assigned to the EMB professor, Economics Professor, or in our case, both instructors review the audio quality.

The final project is due on the 13th weeks of the semester. Both instructors review the submissions and select which videos will be presented at the screening. We have experimented with restricting the
number of videos that are part of the final event. We have found that, if time isn’t a constraint, including all submissions makes the event a better experience as measured by attendance and participation. The screening event is scheduled during the final (14th) week of classes.

In the appendix we provide instructors with a rubric and example of an assignment timeline. Instructors looking to implement the Econ Beats project can adjust the due dates or create more low-stake assignments to help their students reach the final projects. Other modifications can also be made. For institutions without an EMB department, Economic instructors can collaborate with Theater, English, Music, or Communication departments. While our collaboration with EMB helps increase the production quality, we postulate that there are gains that can be experienced by working with other departments. If no other collaboration opportunities exist it is possible for economics students to create their own videos. In earlier versions of the assignment we attempted the project with economics students only. Students were able to complete the assignment, however, the production quality was much lower. 3

The Screening Event

The highlight and what differentiates Econ Beats from other music and arts projects in economics is the end of semester public screening. It is important to celebrate the students work. This aspect makes the assignment more engaging, and attracts other students to economics. The event is open to the public, we have hosted students from NKU, faculty members, high school students from the surrounding area, and other members of the community. The Econ Beats event is a fun and enjoyable experience for students, their friends, and families. We were able to award the top 3 projects with cash prizes.4 The top 3 videos are selected by popular vote based on the audience. Audience selection allows everyone to participate and engage in reviewing economic ideas. The order of the prizes is determined by a faculty member from economics, an EMB faculty, and a former winner of Econ Beats. Cash prizes totaled $1,000 per semester. However, in other versions of this contest, we were able to host a successful Econ Beats event with less than $100 in awards.

Examples of Student Projects

To best appreciate the production, and economic content of Econ Beats assignments is to view some examples. We provide three examples of the level Economic and students production quality that students produce.

The first video is called Unemployment Journey. It provides viewers with two remakes of popular songs by the group Journey. The video discusses the different type of unemployment in the economy and issues with measuring unemployment. Economic instructors looking to introduce or review the unemployment chapter can use this student produced video to reinforce the topic of unemployment.

The video Econ Bling can be used when discussing the loanable funds market. The remake of Drake’s Hotline Bling was popular with students during the spring 2016 semester. While the amount of economic content isn’t as abundant as in Unemployment Journey, instructors can use Econ Bling for active learning activities as suggested in (Lawson, Hall, and Mateer 2008).

The song Agents of Unemployment is a remake of Justin Bieber’s Love Yourself. This video also discusses unemployment types and unemployment measurement. Students approach the discussion of unemployment in a different context than in Unemployment Journey. The varied ways in which students’ express common economic concepts is what makes Econ Beats an interesting assignment. The examples provided show how Econ Beats allows the students to creatively express their understanding of complex concepts.

Conclusion

We provide instructors with an example of how to incorporate a semester-long, interdisciplinary, project based learning assignment. Business instructors have shied away from adjusting their curriculum, although studies have found that students need to be better prepared for today’s job market. We believe that

3 For economics only assignments, instructors can assign poetry, or limericks. For other examples of how to use the Arts in Economics courses see Al-Bahrani, Abdullah. A., Kim Holder, Darshak Patel, & Jadrian Wooten (2016). Art of Econ: Incorporating the Arts through Active Learning Assignments in Principles Courses. Journal of Economics and Finance Education. 15(2),1-17

4 Sponsorship was provided by Centsai.com
Econ Beats is an assignment that prepares students for real-life work environment and helps students learn and retain economics better. The Econ Beats assignment helps students learn how to manage a long term project, work in groups, think creatively, and communicate effectively, while exposing students to economic concepts. The entertainment factor of the end of semester screening also helps engage the university community, raising awareness of economics and economic concepts. Future research in this area should evaluate the efficacy of Econ Beats type assignments in increasing student learning.

References


Appendix
Econ Beats

Description, Timeline, and Rubric

Any popular song may be chosen, but your song choice is subject to my approval. Song lyrics must be substantially rewritten (i.e. your own original work). No unaccompanied versions are accepted. The vocalist must be singing to recorded music and you must be able to hear the music on your video. The vocalist does not have to be a class member. When you post to YouTube, your lyrics MUST be clearly visible and readable. The lyrics must be part of your original file and not just utilizing the captions function of YouTube. Your video MUST include the following tags: “NKUEcon”, “Fall 2016”, and “Econ Beats”. You will submit your assignment in two ways

A) Upload your video on YouTube and submit your link on Blackboard via the assignment folder.
B) Submit the original digital file for my records.
C) See below rubric for more information

Below is the grading scale and some samples of past submissions so you know what I am looking for in your project. Think outside the box and remember that Economics is more than just about making money! Think about all the topics we have covered in class and relate them to the world around you (If “making money” is the central theme of your video, you will only receive 1 point for creativity). Don’t forget to have fun and let the best team win! Good Luck!

1. You will receive 20 total points for the six assignments leading up to the video submission. The Econ Beats video will be the seventh assignment and worth 80 points.

<table>
<thead>
<tr>
<th>Assignment</th>
<th>Points</th>
<th>Due</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st Assignment- Econ Beats Survey and group assignments (2 points)</td>
<td>2</td>
<td>Sept 2nd</td>
</tr>
<tr>
<td>2nd Assignment- First group meeting (2 points)</td>
<td>2</td>
<td>Sept 9th</td>
</tr>
<tr>
<td>You will receive all 2 points if you submit the name of your group members by the due date. As a group you must watch last year’s videos and pick your favorite video. Include that in your summary.</td>
<td>2</td>
<td>Sept 9th</td>
</tr>
<tr>
<td>3rd Assignment- Meeting with your EMB group (6 points)</td>
<td>6</td>
<td>Sept 23rd</td>
</tr>
<tr>
<td>Your assignment is to submit minutes from your meeting. Indicating the name of the members that attended, location and date of meeting, and a paragraph summary of your discussion. One submission per group.</td>
<td>6</td>
<td>Sept 23rd</td>
</tr>
<tr>
<td>4th Assignment- Song Selection (2 points).</td>
<td>2</td>
<td>Nov 4th</td>
</tr>
<tr>
<td>Provide the name of the song, and a YouTube link to the song.</td>
<td>2</td>
<td>Nov 4th</td>
</tr>
<tr>
<td>5th Assignment- Lyrics and Audio Due (2 points)</td>
<td>2</td>
<td>Nov 11th</td>
</tr>
<tr>
<td>You will be graded on the quality of the audio, originality of the lyrics, and the economic content of the lyrics. You are responsible for verifying that the lyrics you are using are acceptable and accurate. Feel free to stop by my office hours and see me if you have any questions</td>
<td>2</td>
<td>Nov 11th</td>
</tr>
<tr>
<td>6th Assignment- Group evaluation form (6 points)</td>
<td>6</td>
<td>Dec 2nd</td>
</tr>
<tr>
<td>7th Assignment- The Econ Beats video (80 points)</td>
<td>80</td>
<td>Dec 2nd</td>
</tr>
<tr>
<td>See below rubric.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2. All Submissions must include the following or the score will be an automatic ZERO:

<table>
<thead>
<tr>
<th>Requirement</th>
<th>Yes</th>
<th>No</th>
</tr>
</thead>
<tbody>
<tr>
<td>A short description explaining what the video is about (2-3 Sentences)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Video embedded subtitles</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Names of group Members</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Digital copy on USB for submission</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Link to the uploaded YouTube video</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3. Your Econ Beats project will be graded on a 80 point scale according to the following table: Here is a playlist from last semester's top 6 videos www.abdullahalbahrani.com/econbeats/

<table>
<thead>
<tr>
<th>Category</th>
<th>Unacceptable</th>
<th>Barely Acceptable</th>
<th>Average</th>
<th>Meets Expectations</th>
<th>Goes beyond expectations</th>
<th>Score Options</th>
</tr>
</thead>
<tbody>
<tr>
<td>Audio Quality</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Video Quality</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Description/Concept</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Economic Concepts/lyrics</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creativity</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Samples of past submissions:
- [https://www.youtube.com/watch?v=Uz33TnPYs0&list=PLMdNY3lwUXWQi6OngYhGoWD5KIQWRT](https://www.youtube.com/watch?v=Uz33TnPYs0&list=PLMdNY3lwUXWQi6OngYhGoWD5KIQWRT)
- [https://youtube.com/watch?v=9yi3j2lwqH8&list=PL65CE3624623304J9&index=3](https://youtube.com/watch?v=9yi3j2lwqH8&list=PL65CE3624623304J9&index=3)
The Relative Value of AER P&P Economic Education Papers

J.R. Clark, Joshua C. Hall, and Ashley S. Harrison

Abstract

The Committee on Economic Education (CEE) has been allocated one session in the American Economic Review Papers and Proceedings (P&P) since 1964. In 2008, the American Economic Association evaluated the allocation of AER Papers and Proceedings sessions to various AEA Committees. In response, the CEE was asked to prepare a one-page rationale for keeping that session. Their response made several important defenses of the session, including that the quality of the papers published in these sessions must speak for itself. In this paper, we evaluate the relative quality of AER P&P papers through citation analysis. Using the Social Science Citation Index, the citation counts of economic education AER P&P papers are compared to other P&P papers.

Introduction

Economic education has played an important role within the American Economic Association (AEA) since the association was formed in 1885. Hinshaw and Siegfried (1991) point out that two of the nine sessions in the 1890 meetings were on the topic of economic education. For the next half-century or so, papers on economic education regularly appeared at the annual meetings. The content of these sessions varied widely, but most focused on what economics was currently being taught in high schools and colleges and how it might be improved (Hinshaw and Siegfried 1991). For example, Professor Folsom (1925) of Sweet Briar College suggested that rapid improvement in the teaching of economics will only occur if economics departments become more like industry and willing to invest in greater organization (sharing of materials, coordination on the basics of what should be taught, etc.) and equipment (standardized ‘scientific’ textbooks, permanent graphs, etc.). Sessions and correspondence on topics related to what was taught and what should be taught in the principles course were most common during this period.

The 1940s and 1950s would see the AEA take on a more formal role with respect to economic education. In 1944, the Executive Committee of the AEA created the Committee on Undergraduate Teaching in Economics and Training of Economists. Within the Committee, there were ten subcommittees working on various topics related to economic education such as economics in schools of business, economics and agricultural education, the undergraduate curriculum, etc. (Hinshaw and Siegfried 1991). This was an enormous undertaking that demonstrates how important the issue was to its membership. Twenty-two consultants were employed on the project and fifty-six members of the AEA participated in this multi-year undertaking.²

In 1955, this ad hoc committee was reorganized as a standing committee named the Committee on Economic Education (CEE). According to Hinshaw and Siegfried, (1991, p. 377), the CEE was created “to serve as a focal point for improving the status of economic education within the field of professional economists, to stimulate and encourage professional work on economic education, and to encourage the preparation of articles about and arrange sessions at the annual AEA meetings on economic education.” The late 1950s saw the CEE undertaking many activities, including organizing sessions on economic education that were subsequently published in the American Economic Review:

---

1 Clark is the Probasco Chair of Free Enterprise at the University of Tennessee at Chattanooga, 313 Fletcher Hall, Dept. 6106, 615 McCallie Avenue, Chattanooga, TN 37403-2598, Probasco@utc.edu. Hall is an Associate Professor of Economics and Director of the Center for Free Enterprise at West Virginia University, 1601 University Ave, PO Box 6025, Morgantown, WV 26506-6025, joshua.hall@mail.wvu.edu. Harrison is the Assistant Director of the Center for Economic Education at the University of Tennessee at Chattanooga, 313 Fletcher Hall, Dept. 6106, 615 McCallie Avenue, Chattanooga, TN 37403-2598, Ashley-Harrison@utc.edu. The authors would like to thank John Breeden and Clint Thompson for their research assistance. They would also like to thank Gail Hoyt, Bill Alpert, and other session participants at the 2015 meetings of the Southern Economic Association. Two anonymous referees provided excellent comments that improved the paper.

2 Further evidence of the importance of this project is given by the fact that two of those involved in the undertaking were prominent members of the profession who would eventually go on to earn Nobel Prizes in Economics, namely George Stigler and T.W. Schultz.
Papers and Proceedings. The CEE was dissolved and reorganized under the same name in 1963, with the charge of being more active in improving economic education at all levels (Hinshaw and Siegfried 1991). The CEE has existed continuously within the AEA structure since that time and since 1964 (with the exception of 1965) has had at least one special session on economic education at the annual meetings. Papers and Proceedings of the American Economic Association that appear every year in the American Economic Review have therefore contained the contents of at least one economic education session since 1964, with the exception of the 1966 volume since there was no economic education session during the 1965 meetings. The AER Papers and Proceedings session dedicated to the CEE was formally established by the AEA Executive Committee at its March 1964 meeting (Executive Committee of the American Economic Association 1965).

In 2007, the Executive Committee of the AEA asked the President to create an ad hoc committee on the Papers and Proceedings. This decision emanated from a previous discussion on the allocation of annual meeting slots and standing ‘property rights’ to publish session papers in the Papers and Proceedings that occurred at the January 2007 Executive Committee meeting in Chicago (Executive Committee of the American Economic Association 2007). At the time, papers from 24 sessions were published in the Papers and Proceedings. In addition to the Richard Ely Lecture and the Committee on Economic Education, the National Economic Association had the right to publish one session in the Papers and Proceedings and the Committee on the Status of Women in the Economics Profession had the right to publish two sessions (Executive Committee of the American Economic Association 2007). Given the scarcity of space within the Papers and Proceedings and anticipated requests for guaranteed slots by the Committee on Economic Statistics and the Committee on the Status of Minority Groups in the Economics Profession for guaranteed slots, the creation of an ad hoc committee to evaluate the distribution and allocation of existing rights seemed appropriate.

The decision to evaluate the practice of allocating Papers and Proceedings led to each AEA Committee being asked to prepare a one-page rationale for maintaining their property rights over an individual session (Committee on Economic Education 2008). At the time, there was concern that any number of possible changes could be on the table. According to the minutes of the Committee on Economic Education (2008), the Ad Hoc Committee on Standing Sessions in the Papers and Proceedings could lead to the diminution or elimination of the right to publish the contents of one session in the proceedings. The CEE’s one page report appeared in Committee on Economic Education (2008) and lists six reasons for the continued inclusion of a CEE session in the Papers and Proceedings. After assessing the evidence, the Executive Committee of the AEA voted that that the CEE would continue to have rights over one Papers and Proceedings session a year (Executive Committee of the American Economic Association 2009). While the CEE still has property rights over one economic education session appearing annually in the Papers and Proceedings thanks to the arguments put forth in Committee on Economic Education (2008), this property right is limited. As part of the report of the Ad Hoc Committee on Standing Sessions in the Papers and Proceedings in 2008, the Executive Committee of the AEA voted to “set a fixed, renewable term for the inclusion of standing sessions not to exceed five years” (Executive Committee of the American Economic Association 2008, p. 571). With the number of papers submitted to the American Economic Association annual meetings continuing to rise and the number of sessions appearing on the annual program, but excluded from the Papers and Proceedings, the relative value of economic education papers appearing in the Papers and Proceedings will likely continue to be debated.

In this paper we seek to add a quantitative perspective to the question of the relative value of economic education papers appearing in the American Economic Review: Papers and Proceedings. Going back at least to the work of Fels (1969), there has been concern that research in economic education was not as “high quality” as in other areas of economic inquiry. Committee on Economic Education (2008, p. 7) alludes to concerns over quality when it states that “Ultimately, the quality of the papers published in these [CEE] sessions must speak for itself.” While quality can be subjectively measured, economists have long used citation counts as a quantitative measure of the quality of a scholar’s research (Hamermesh et al. 1982; Diamond Jr. 1986; Hamermesh and Pfann 2012). While this “revealed preference” approach has limitations and cannot speak to the other benefits of the CEE Papers and Proceedings sessions laid out in Committee on Economic Education (2008), our findings reflect favorably on the inclusion of these sessions that year.

3 Hinshaw and Siegfried (1991) do not mention what happened in 1965 and we have been unable to determine why there was no economic education session that year.

4 In short, the six reasons are: 1) demand for economic education research and sessions has increased considerably since 1965; 2) these sessions and papers are often commissioned as a service to the discipline of economics; 3) the economic education P&P papers are widely read and cited; 4) the P&P session helps recruit top economists to serve on the CEE; 5) the P&P session is the majority of the support the AEA gives to economic education efforts; and 6) the majority of economic education papers in the P&P are high quality. For the full argument, see Committee on Economic Education (2008).

5 The Executive Committee also voted to expand the number of sessions to 34 and to allocate sessions to all current committees and to the Committee on Economic Statistics and the Committee on the Status of Minority Groups in the Economics Profession. For the entire discussion of the Ad Hoc Committee, see (Executive Committee of the American Economic Association 2009, p. 650).
papers. To preview our findings, we find that the top ten economic education articles are better cited than 75 percent of all non-economic education papers in the Papers and Proceedings. We also find no statistical difference between the number of economic education P&P papers and non-economic P&P papers that have zero citations.

Empirical Approach, Data, and Results

The use of citations to assess the quality of a particular piece of research in economics, and consequently the author of the research and their academic home, has a long history, but took off in earnest with the publication of the Social Science Citation (SSCI) in 1973. The SSCI is a database of scholarly literature that includes citation data to other journals that are catalogued within the SSCI. Citations reports produced using the index are used to evaluate research productivity and personnel decisions in a number of departments (Klein et al. 2004a). Researchers have used SSCI data in a variety of ways, from ranking journals and departments (Coupé 2003) to evaluating intellectual influence (Beaulier and Hall 2009; Boettke et al. 2012).

Our purpose here is to evaluate the relative quality of the economic education papers published in the Papers and Proceedings issue of the American Economic Review using citation data from the SSCI. We employ the SSCI data instead of data from Scopus or Google Scholar. The limited amount of research comparing results across the three indices finds that Google Scholar is systematically different than Scopus or SSCI (Bar-Ilan 2008). Google Scholar, for example, often picks up more citations by being less selective. Google Scholar does not allow for the systematic exclusion of self-citations. Scopus, while having an extensive coverage and being similar to SSCI post-1996, has no reference data before 1996 (Bar-Ilan 2010).

Our sample period begins in 1964 to correspond with the AEA Executive Committee decision to allocate one Papers and Proceedings session to the CEE. Our measure of article quality is the average number of citations—an economic education article received per year from the time of publication until the time of data collection in April of 2015. For each year of the Papers and Proceedings, we identified the CEE sponsored sessions and calculated the average citations per year for all research articles in the proceedings. In order to give recent articles time to garner citations, we stop our analysis with the 2011 Papers and Proceedings. We only measure citations to other journals in the SSCI. Over this time period there were 3,494 total research articles in the Paper and Proceedings, 130 of which were categorized as economic education.

Figure 1 illustrates the evolution of citations of economic education articles in the Papers and Proceedings from 1965 to 2011. It is important to note that the citations for each session have been averaged within each year in this figure, obscuring that some articles receive more citations than others. The number of citations per year per article seems to be increasing. To the extent that citations are an accurate measure of quality, this upward trend in citations could reflect increased paper quality. The data seem to reflect what others have observed about economic education since the criticism levied by Fels (1969), namely that the quality of economic education articles seems to be on the rise (Allgood et al. 2015).

Two caveats are in order. First, while increasing, the average number of citations per year are relatively low. Without a comparison group, these numbers only tell us that the number of citations garnered per year per CEE-sponsored session papers are rising. This could reflect that these articles are of increased quality, or merely reflect underlying trends in citation practices. For example, Laband and Tollison (2003) find that the number of references per paper, and thus citations, are increasing over time. Second, these data could also reflect an expansion in the number of journals included in the SSCI over time.

---

6 The SSCI was first published in 1973 (Klein et al. 2004b). Its current publisher, Thompson Reuters, has subsequently back filled social science information to 1900. Gerrity and McKenzie (1978) provides an illuminating discussion of the use of citations to assess the quality of research.
7 Many economic education papers are written primarily to disseminate teaching innovations or advice. See, for example, Hall (2005), Lawson et al. (2008), Gillis and Hall (2010), and Chamlee-Wright et al. (2017). These articles can have an influence on the practice of teaching economics without every garnering a citation. This is an important caveat to our citation-based approach, and one we deal with more extensively later in this section.
8 While this is not by itself problematic, Google Scholar sometimes double counts citations by citing both the working paper version of a paper and the published version. Also, non-research such as syllabi and magazine articles are sometimes automatically included.
9 We did start out collecting data from Scopus for the post-1996 period. In our experience, however, we could not locate several Papers and Proceedings articles in Scopus. We had no such problem with the SSCI.
10 From this point onward we use economic education articles synonymously with the CEE sponsored session as we observed zero economic education articles appearing the Papers and Proceedings that were not part of a CEE session.
11 In practice, this meant we excludes all proceedings material related to the business of the American Economic Association such as committee reports.
12 The number of journals in the SSCI has increased since 1973, but we were unable to find exact numbers by year.
Figure 1. Average Yearly Citations for Economic Education Articles in AER P&P

Note: Source of citation data is the Social Science Citation Index. Average citations per year are calculated from the time of publication through April 2015. The 1965 American Economic Association meetings did not have an economic education session. Therefore, the 1966 Papers and Proceedings contained zero economic education articles.

Figure 2 provides some insight into these caveats. It shows the average number of citations per non-economic education papers in the Papers and Proceedings. The figure shows a similar upward trend in the average number of citations per year. Even if there is citation decay, these numbers seem to suggest that average citations per article per year are rising over time, probably due to an increased number of references per article and the increased number of journals included in the SSCI. While the average number of citations are increasing over time for economic education articles and general research articles, the average number of citations for non-economic education articles are generally higher. This is not surprising given that most of the journals that are likely to cite an economic education article are not included within the SSCI. The only field journal included in the SSCI over this time period is Journal of Economic Education and the only journals that even occasionally publish economic education articles are the Southern Economic Journal, Economic Inquiry, Economics of Education Review, and the occasional article in Journal of Economic Perspectives and Journal of Economic Literature. Like Weintraub (2006) noted with History of Political Economy, it is difficult to have high citation numbers in the SSCI when the journals that cite you are not included in the SSCI.

The mean number of annual citations per economic education article over the period is 0.60, while the mean number of annual citations per non-economic education articles was 1.42. A Welch’s t-test of the sample means assuming unequal variances shows a statistically significant difference at the one-percent level between these two means. This is not surprising given that many of the journals likely to cite the CEE articles in the Papers and Proceedings are not listed in the SSCI. In addition, some of the economic education articles, while effective in conveying lessons to economic educators, are unlikely to garner a large number of future citations as they are not “research” per se. A good example of an article of this type is Hamermesh (2002). An overview of the author’s tricks for teaching microeconomics well, the article has garnered zero SSCI citations in thirteen years. These tips, however,

---

13 It should be noted that Economic Inquiry ended its economic education section “Teaching Tools” over a decade ago.
15 Welch’s t-test is used to test the hypothesis that two populations have equal means when the two samples have unequal variances and unequal sample sizes.
16 The article received 10 Google Scholar citations over that period.
could have helped numerous beginning economic educators over the years, highlight the difference between quality as an input into others’ teaching and quality in terms of an input into others’ research.

Figure 2. Average Yearly Citations for Non-Economic Education Articles in AER P&P

Even measured in terms of being an input into the research of others, it is unclear if comparing average citations rates over the sample period is the proper measure. After all, the non-economic education articles have a number of “superstar articles” that garner in excess of 20 citations annually. Table 1 lists the ten articles with the highest average annual citations rates over this period. Surprisingly, only one article is a Richard T. Ely Lecture (Stern 2008).

Table 1: Ten Best Cited Non-Economic Education P&P Papers, 1965-2011

<table>
<thead>
<tr>
<th>Article Title</th>
<th>Author(s)</th>
<th>Year</th>
<th>Avg. Citations Per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agency Costs of Free Cash Flow, Corporate Finance, and Takeovers</td>
<td>Jensen</td>
<td>1986</td>
<td>110.0</td>
</tr>
<tr>
<td>Clio and the Economics of QWERTY</td>
<td>David</td>
<td>1985</td>
<td>51.8</td>
</tr>
<tr>
<td>In Search of Homo Economics: Behavioral Experiments in 15 Small-Scale Societies</td>
<td>Henrich et al.</td>
<td>2001</td>
<td>41.0</td>
</tr>
<tr>
<td>Tunneling</td>
<td>Johnson et al.</td>
<td>2000</td>
<td>31.2</td>
</tr>
<tr>
<td>Toward a Theory of Property Rights</td>
<td>Demsetz</td>
<td>1967</td>
<td>29.3</td>
</tr>
<tr>
<td>The Economics of Climate Change</td>
<td>Stern</td>
<td>2008</td>
<td>26.9</td>
</tr>
<tr>
<td>I Just Ran Two Million Regressions</td>
<td>Salai-i-Martin</td>
<td>1997</td>
<td>26.4</td>
</tr>
<tr>
<td>Inductive Reasoning and Bounded Rationality</td>
<td>Arthur</td>
<td>1994</td>
<td>26.4</td>
</tr>
<tr>
<td>Trust in Large Organizations</td>
<td>La Porta et al.</td>
<td>1997</td>
<td>25.3</td>
</tr>
<tr>
<td>The Technology of Skill Formation</td>
<td>Cunha and Heckman</td>
<td>2007</td>
<td>24.6</td>
</tr>
</tbody>
</table>

The top ten economic education articles are listed in Table 2. While influential, they are not going to influence scholarship in the same way that Demsetz (1967) started an entire area of inquiry into the economics of property rights. The most cited CEE article is John Taylor’s article on teaching modern macroeconomics (Taylor, 2000). Economic educators will certainly recognize the articles detailing the extent of “chalk and talk” by Becker and Watts.
(1996) and Becker and Watts (2001). These two articles by Becker and Watts are examples of research that not only affects what goes on inside the classroom, but stimulates additional research into alternative teaching approaches (Hoyt and McGoldrick 2012; Hall and Podemska-Mikluch 2015).

Table 2: Ten Best Cited Economic Education P&P Papers, 1965-2011

<table>
<thead>
<tr>
<th>Article Title</th>
<th>Author(s)</th>
<th>Year</th>
<th>Avg. Citations Per Year</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Modern Macroeconomics at the Principles Level</td>
<td>Taylor</td>
<td>2000</td>
<td>4.7</td>
</tr>
<tr>
<td>Chalk and Talk: A National Survey on Teaching Undergraduate Economics</td>
<td>Becker and Watts</td>
<td>1996</td>
<td>4.3</td>
</tr>
<tr>
<td>Can Web Courses Replace the Classroom in Principles of Microeconomics?</td>
<td>Brown and Liedholm</td>
<td>2002</td>
<td>3.9</td>
</tr>
<tr>
<td>Teaching Economics at the Start of the 21st Century: Still Chalk-and-Talk</td>
<td>Becker and Watts</td>
<td>2001</td>
<td>3.4</td>
</tr>
<tr>
<td>The Effect of National Standards and Curriculum-Based Exams on Achievement</td>
<td>Bishop</td>
<td>1997</td>
<td>2.7</td>
</tr>
<tr>
<td>The Effects of Attendance on Student Learning in Principles of Economics</td>
<td>Durden and Ellis</td>
<td>1995</td>
<td>2.1</td>
</tr>
<tr>
<td>The Lake Wobegon Effect in Student Self-Reported Data</td>
<td>Maxwell and Lopus</td>
<td>1994</td>
<td>1.8</td>
</tr>
<tr>
<td>What Students Remember and Say about College Economics Years Later</td>
<td>Allgood et al.</td>
<td>2004</td>
<td>1.7</td>
</tr>
<tr>
<td>Efficiency in the Use of Technology in Economic Education: Some Preliminary Results</td>
<td>Sosin et al.</td>
<td>2001</td>
<td>1.6</td>
</tr>
<tr>
<td>How Departments of Economics Evaluate Teaching</td>
<td>Becker and Watts</td>
<td>1999</td>
<td>1.6</td>
</tr>
</tbody>
</table>

The articles listed in Table 2 are better cited than 75 percent of all non-economic education articles in the Papers and Proceedings. Clearly the best economic education papers “hold their own” when compared to the average paper in the Papers and Proceedings. In terms of advancing knowledge, perhaps the appropriate measure is not the citations garnered at the top, but the number of “dry holes.” Laband and Tollison (2003) demonstrate that a surprising number of articles in top journals are “dry holes” in that they generate little to no additional citations. Looking at the non-economic education articles, 190 of them have garnered no citations. That is 5.6 percent of the entire sample of non-economic education Papers and Proceedings articles. For the economic education articles there were 9 articles that received zero citations per year, for a dry hole percentage of 7.6 percent. Using a two-sample test of proportions, we cannot reject the null that these proportions of dry holes are equal.

Table 3: Nine Economic Education Articles with Zero SSCI Citations

<table>
<thead>
<tr>
<th>Article Title</th>
<th>Author(s)</th>
<th>Year</th>
<th>Total Google Scholar Citations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Teaching Economics to Black Students</td>
<td>Lloyd</td>
<td>1971</td>
<td>1</td>
</tr>
<tr>
<td>Videotaped Dialogues in Economics</td>
<td>Castro</td>
<td>1971</td>
<td>0</td>
</tr>
<tr>
<td>The Teacher Training Program for New Ph.D.s</td>
<td>Lewis and Becker</td>
<td>1976</td>
<td>4</td>
</tr>
<tr>
<td>On Teaching Teachers to Teach</td>
<td>Long</td>
<td>1976</td>
<td>1</td>
</tr>
<tr>
<td>One Participant’s View of the Teacher Training Program</td>
<td>Hansen</td>
<td>1976</td>
<td>1</td>
</tr>
<tr>
<td>International Perspectives in Undergraduate Education</td>
<td>Stiglitz</td>
<td>1993</td>
<td>14</td>
</tr>
<tr>
<td>Microeconomic Principles Teaching Tricks</td>
<td>Hamermesh</td>
<td>2002</td>
<td>10</td>
</tr>
<tr>
<td>The Effectiveness of Peer Tutoring on Student Achievement at the University Level</td>
<td>Munley et al.</td>
<td>2010</td>
<td>8</td>
</tr>
<tr>
<td>The Efficacy of Collaborative Learning Recitation Sessions on Student Outcomes</td>
<td>Huynh et al.</td>
<td>2010</td>
<td>11</td>
</tr>
</tbody>
</table>

Table 3 lists the nine economic education articles without a citation. Looking at the economic education dry holes, it provides some evidence that economic education dry holes are different than non-economic education dry holes. There are several things to note from the table. First, the economic education dry holes seem to be concentrated...
in a handful of years (1971, 1976, 2010). The same is not true of the 190 non-economic education articles in the Papers and Proceedings, which are spread out from 1965-2011. Second, the articles garnering no citations are clearly written with the purpose of disseminating information, not spurring new research in economic education. The “dry holes” from the 1970s would seem to clearly fall into that category. Third, this list includes a Nobel Laureate in economics (Stiglitz) and a well-known labor economist (Hamermesh), highlighting that it is the purpose behind economic education articles that seems to influence further citations. Stiglitz and Hamermesh are writing as teachers of economics, not researchers studying the teaching of economics. It is no surprise then that their papers have garnered zero SSCI citations. Finally, the fourth column shows the Google Scholar citations for these articles as of November 2015. Looking at the most recent articles, it can be seen that these research-oriented articles are garnering citations, just not in journals in the SSCI. This further highlights the issue raised by Weintraub regarding citations to papers in fields not well-represented in the SSCI.

Concluding Thoughts

Overall, economic education papers in the Papers and Proceedings receive fewer citations in the SSCI than non-economic education papers. Further work needs to be done to see the extent to which this is influenced by the fact that, with the exception of The Journal of Economic Education, no other economic education journal is in the SSCI. Regardless of the built-in bias against economic education articles inherent in our approach, we feel that the results reflect favorably on the CEE articles included in the Papers and Proceedings. While the average citations per year for economic education articles are lower than for non-economic education articles, the difference is not large and can be partly explained by the different missions of the CEE articles and the structure of the SSCI. When looking at the top cited articles in economic education, we find that they receive more citations per year than 75 percent of the non-economic education articles. Looking beyond the top articles, we find no statistically significant difference between the proportion of economic education articles and non-economic education articles garnering zero citations over the nearly forty-year period of our sample. Finally, we find that the citations to CEE articles in the Papers and Proceedings is increasing over time and the number of “dry holes” is declining.

Given the statistically significant difference in mean number of citations per year between economic education and non-economic education articles, we might be accused of being too optimistic regarding the relative value of CEE articles. However, our view reflects that economic education articles appearing in the Papers and Proceedings are chosen with several purposes in mind as pointed out in Committee on Economic Education (2008). For example, many of the papers are chosen as a service to the profession or to entice prominent economists to put forth their views on economic education. Sacrificing these benefits in order to garner more citations would be, in our view, short-sighted. Given the multiple margins upon which the CEE is optimizing, we view these results as being a confirmation that they are meeting their stated goal of “choosing papers for the Proceedings session that will be widely read and cited in the profession” (Committee on Economic Education 2008, p. 7).

References


It should also be noted that these articles are unlikely to pick up citations in the future as they refer to technology and programs that are no longer in use.


Teaching the Paradox of Thrift with a Water Rescue Analogy

Ben L. Kyer\textsuperscript{1} and Gary E. Maggs\textsuperscript{2,3}

Abstract

This paper compares the familiar paradox of thrift in macroeconomics with an attempted water rescue of a distressed swimmer. We find two primary similarities. First, while the paradox of thrift demonstrates that an increased desire to save more dollars results in fewer dollars actually saved, the water rescue scenario shows that the desire to save one life may result in two lives actually lost. Second, the respective roles of the saving and investment functions in the paradox of thrift are essentially paralleled by the rescuer and the rescuee in the water rescue analogy.

Introduction

The paradox of thrift remains a viable topic and discussed phenomenon in macroeconomics, appearing still in several principles and intermediate textbooks.\textsuperscript{4} Samuelson (1948) is usually credited with both coining the term ‘paradox of thrift’ and first demonstrating it graphically.\textsuperscript{5} The idea and intuition of this paradox, however, is found in Keynes (1936, page 111) where he wrote “Thus, even if it is the case that a rise in the rate of interest would cause the community to save more \textit{out of a given income}, we can be quite sure that a rise in the rate of interest (assuming no favourable change in the demand-schedule for investment) will decrease the actual aggregate of savings.” Keynes in turn (1936, page 361) cited Mandeville’s passage in \textit{Fable of the Bees}: “…\textit{Saving}, is in private families the most certain method to increase an estate, so some imagine that, whether a country be barren or fruitful, the same method if generally pursued (which they think practicable) will have the same effect upon a whole nation, and that, for example, the English might be much richer than they are, if they would be as frugal as some of their neighbours. This, I think, is an error.”

Regardless of its true origination and while it is of course easy to show with a diagram, we find that many students more readily understand the paradox of thrift as it might be portrayed with a non-economic analogy. The analogy we use frequently in our classrooms with some success and offer here but have not found in any previous literature, neither textbook nor journal article, is that of an attempted water rescue of a struggling or drowning swimmer. Our pedagogical approach to the paradox of thrift detailed below follows in the tradition of other shorter economic education papers which demonstrate various techniques to teach challenging economic concepts.\textsuperscript{6}

The Analogy

We begin with a formal definition of the paradox of thrift, i.e., an increase in the desire to save results in decreased actual saving, followed closely by the less formal description that when people try to save more dollars they end up saving fewer. These verbal definitions are followed by the traditional graphical demonstration of this paradox with the simple Keynesian cross diagram where both saving and investment depend positively on real gross domestic product or aggregate real income. The original equilibria are shown and then disturbed by an increased desire to save with the resulting upward shift of the saving function. We

\textsuperscript{1} The Benjamin Wall Ingram, III, Professor of Economics, Francis Marion University, PO Box 100547, Florence, SC, 20501. Email: BKyer@FMarion.edu.
\textsuperscript{2} Professor of Economics, St. John Fisher College, 3690 East Avenue, Rochester, NY, 14618. Email: GMaggs@SJFC.edu.
\textsuperscript{3} The authors thank the Editor of this \textit{Journal} for helpful comments and suggestions on an earlier version of this paper.
\textsuperscript{4} See for example Blanchard and Johnson page 59, Colander page 568, Krugman and Wells pages 599 and 765, and McConnell et al page 554.
\textsuperscript{5} Recent editions of this text, now Samuelson and Nordhaus, however, no longer mention the paradox of thrift.
\textsuperscript{6} See for example Asarta and Mixon, Hall, Kassens, and Sobel and Hall.
show the aggregate surplus created by this disturbance and explain following Keynesian theory the resulting decrease of real GDP. This decline in real income causes then investment spending to decrease and, in order to restore equilibrium, saving must also decrease. Thus the paradox that a desire to save more dollars results in fewer dollars actually saved. The point which we particularly emphasize here is that induced investment spending is the active or operative force that has pulled down or dragged down or caused less actual saving. To contrast and additionally stress this conclusion, we remind students of the result from an earlier lecture that an increased desire to save does not decrease actual saving when investment spending is autonomous.

At this juncture in the classroom discussion we ask our students if any of them have had any experience with water rescues, such as lifeguarding at a swimming pool or a beach. We then explain that the hierarchical order of the alternative water rescue techniques is “reach, throw, row, and go” and define each of these words, where “go” means to swim to the distressed individual in an attempt to save their life. While each of the four techniques involves some degree of risk, swimming to the individual is inherently the most dangerous and therefore should be the last resort since the distressed person, fearful for his or her life and likely quite strong because of the increased adrenaline, may be thrashing frantically, may clutch onto and/or attempt to climb the rescuer or lifeguard, and force them both down under the water to drown. In other words, the rescuee may pull down or drag down the rescuer as induced investment pulled down saving in the paradox of thrift. The tragic result and the sad paradox here is that the attempt to save one life results in two lives actually lost.

Conclusion

The parallels between the two situations are straightforward and we summarize them for our classes. First, while an increased desire to save more dollars in the paradox of thrift results in fewer dollars actually saved, the attempt to save one life in the water rescue scenario may result in two lives actually lost. In short then, the paradox of thrift is concerned with dollars whereas the paradox of the water rescue is focused on lives. Further, if this rescue result for water is extended to other failed rescue attempts, such as fires and mines, we may refer to this phenomenon generally as the paradox of rescue.

The second parallel between the two paradoxes concerns not the result but the actors or agents. In particular, the rescuer and rescuee in the water milieu assume essentially the same roles as the saving and investment functions respectively in the thrift case. Once again, the key distinction is that of lives rather than dollars.

References


A Class Activity on the Income and Substitution Effects of a Price Change

Philip Sirianni

Abstract

This paper offers a class activity in which students’ own choices are used to isolate the income and substitution effects of a price change. The exercise is self-contained and requires minimal setup costs for instructors, as a handout outlining the steps students take is provided. The exercise may serve as a useful substitute for or complement to traditional lecturing. The paper includes a discussion of how to assess the effectiveness and students’ perceptions of the activity in one’s own course, and concludes by offering potential variations on the exercise.

Introduction

In my discussions with instructors who have taught microeconomics at the undergraduate level, we generally concur that one of the more challenging topics for students to grasp is the separation of the income and substitution effects of a price change. Some students seem to commit to memory the graphical construction of this decomposition in the hopes that they will be asked to reproduce one of the six possible combinations on an exam; that is, that they will be asked to draw the separation of the two effects for a price increase (or decrease) for a normal, inferior, or (if the instructor is feeling particularly nasty when writing the exam) Giffen good, with little to no understanding of the underlying concepts. Given that this is an essential analytical tool for understanding what is at the core of demand, it is important for students to understand not only the mechanics but even more importantly the intuition behind this abstraction. The purpose of this paper is to offer an alternative to the textbook or chalk-and-talk approach to presenting these topics. This alternative involves a simple class exercise in which the students themselves make choices that reveal their own preferences and lead to the identification of the two effects.

Related Literature

Becker (1997) surveys the state of undergraduate teaching and finds that as institutions began to add and prioritize graduate programs, undergraduate students suffered, particularly in economics. He argues that courses are often poorly taught, and suggests alternatives to chalk-and-talk that may positively affect student learning outcomes. In a seminal symposium paper, Holt (1999) argues that more emphasis should be placed on classroom games and activities in economics courses. Since then, numerous textbooks and journal articles have been published that offer a variety of experiments to economics instructors. A common theme is that the student actually participates in economic phenomena. Kaplan and Balkenborg (2010) suggest that experiments can most easily be introduced in intermediate microeconomics courses, where they may be used to illustrate such concepts as competitive equilibria, supply and demand, price ceilings and floors, and taxes, as in Ruffle (2003, 2005). Nevertheless, Watts and Schaur (2011) find that

1 Assistant Professor of Economics, Department of Economics, Finance, and Accounting, State University of New York at Oneonta, 108 Ravine Parkway, Oneonta, NY 13820, philip.sirianni@oneonta.edu. I thank participants at the Southern Economic Association Annual Meeting, the New York State Economics Association Annual Conference, and two anonymous referees for their comments and suggestions for improving the paper. Most of all, I thank my Intermediate Microeconomics students from the spring of 2016 for participating in the exercise and its assessment. (SUNY Oneonta IRB Application #2016-14.)

experiments are mainly used in introductory courses, and that chalk-and-talk is still the most widely utilized instructional approach, a decade after Becker and Watts (2001) concluded likewise.

Of course, the key question from a research perspective is not necessarily how many instructors have adopted alternative pedagogical approaches, but whether experiments, activities, and games are more, as, or less effective than traditional approaches. Results have been mixed. Fels (1993) claims that devoting just one class to an experiment is not likely to affect student learning, but acknowledges that this is probably true for any individual class period regardless of whether or not an experiment was conducted in it. Cardell et al. (1996) find no positive association with student performance and enrollment in lecture-laboratory sections of courses when compared to lecture-discussion sections. However, Emerson and Taylor (2004) do find evidence that experiments affect student learning outcomes when used over the course of the semester. Emerson and English (2016) argue that five to seven experiments are optimal, and that there are diminishing returns to using them. Durham et al. (2007) find that there is variation in effectiveness which depends largely on the topic. Not surprisingly, the authors conclude that exercises are more useful for more abstract topics.

It may also be that class activities are simply more enjoyable for students than listening to lectures. Dickie (2006) finds that a significant proportion of students prefer exercises to lecturing, Durham et al. (2007) observe a shift in student perceptions (positively) of economics in general when exercises are used, and Becker and Watts (2001) suggest that class activities have a positive effect on teaching evaluations.

Whether the value of class activities is derived from improvements in learning outcomes and retention or from entertainment value (or both), there are some important guidelines which instructors should follow when offering specific activities for use by others. Such guidelines are well-documented in Hazlett (2006), Noussair and Walker (1998), and particularly Emerson (2014). Among them are that any new exercise being presented should be easy to replicate; include all necessary handouts/instructions; include suggestions on how to avoid or deal with common problems that may arise during the exercise; contain a sample student outcome; discuss how to assess the exercise; and summarize students’ perceptions upon completion. I attempt to abide by all these precepts to the extent possible in the activity and in its presentation herein.

The Exercise

Motivation

I start with a brief discussion (no notes on the board – just a casual conversation with copious amounts of hand-waiving) of the basis for learning this analytical tool. I ask the class to consider a health-conscious consumer who wants to buy some fruit. Plums and peaches give her satisfaction, namely because of the vitamins that she knows she will get from them. She also desires some variety, so she plans to put together a particular combination of these two fruits that will make her the happiest.3 At the fruit stand near her apartment, she assembles a basket of plums and peaches and brings it to the counter to pay. It is there that she is informed by the clerk that plums are actually half price; he has not put the sign up yet. Because the conditions under which she assembled her basket have changed, our health-conscious consumer decides to rethink her original plan. She could potentially achieve the same vitamin target by replacing some peaches with plums, since plums are now relatively cheaper. If she does this, the basket will not only cost less but she will be just as satisfied with it. The number of plums added in this case is of course the pure substitution effect.

I then ask the class to recall the consumer’s values, and to consider if she would want to maintain her original vitamin target, thereby leaving some potential satisfaction on the table. Astute students will indicate that she does not need to remove any peaches from the basket; she could simply buy twice as many plums as she had originally planned. In fact, it might even be feasible and desirable for her to add more plums and peaches to the basket. The bottom line is that whatever she decides to do will depend on her (1) preferences, (2) the relative prices, (3) how much money she has, and (4) whether plums are a normal or inferior good. I have found that (4) is the least intuitive for many students, and communicate the income effect this way: The price decrease makes our health-conscious consumer “feel” richer even though she does not actually have more money; therefore, if plums are a normal good, this increase in purchasing power (independent of the change in price) induces her to buy more plums, whereas if they are inferior, she

3 I point out how this relates to a declining marginal rate of substitution.
would buy less. Lastly, I define the terms formally and point out that the substitution effect has nothing to do with whether the good is normal or inferior, that the direction of the two effects is the same for a normal good but opposite for an inferior good, and that the total effect (from which the demand curve may be derived) is the sum of the two effects.

Steps

Once the motivation for the problem is clear, we move on to the exercise, which can be completed in approximately one hour.\(^4\) It may also be helpful to devote some class time (approximately 15 minutes) to reflection on the exercise, and to ask the students to consider other possible outcomes either on their own, in groups, or as a class. On the day the activity is held, students should arrive to class with (or be provided) graph paper, pencils, and rulers. This exercise can be done in several ways and with various choices for the two goods. Here, students are assigned a particular scenario: They have a certain amount of income to allocate to the purchase of on-demand movies from their cable provider (good X) and slices of their favorite type of pizza (good Y) over the course of a weekend. I ask them to assume that if they do not eat, they will starve, and if they are not entertained, they will be consumed with boredom.\(^5\) The prices of movies and slices are also given.

At this point, the students draw their budget line on a diagram, and then take a moment to consider what combination of slices and movies they personally would choose for themselves to maximize their utility. They mark this combination on their diagram as bundle A, and visualize (but do not yet draw) the indifference curve corresponding to the maximum utility level. This is noted as utility level \(U_1\). Now we suppose that the cable provider offers a discount of 50\% so that the price of movies will be cut in half over the weekend. The students draw the new budget line, and choose their new combination of slices and movies (label it bundle C). A new (higher) indifference curve is drawn and marked as \(U_2\). The change in the quantity of movies purchased is marked on their diagram as the Total Effect. Students can also draw the corresponding segment of their demand curve for movies at this point.

We now get to the heart of the exercise. The students chose bundles A and C with a certain (same) amount of income. We now begin to take away income from them incrementally (thus removing the effect of feeling richer), but allow the price of movies to remain at the new (50\% off) level. Furthermore, students are reminded of the basic axioms of consumer theory, and are asked to keep them intact:

1. Completeness: For any two bundles \((A \text{ and } B)\), either \(A\) is preferred to \(B\), \(B\) is preferred to \(A\), or \(A\) and \(B\) are equally preferred.
2. Transitivity: For any three bundles \((A, B, \text{ and } C)\), if \(A\) is preferred to \(B\) and \(B\) is preferred to \(C\), then \(A\) is preferred to \(C\).
3. Non-saturation: All else equal, more of either good is always preferred to less of it.

If any of these assumptions are violated at any point during the exercise, the student will be asked to reconsider his or her income allocation. Instructors may wish to give hints, a few of which are provided directly on the handout in Appendix A and discussed in the example of the next subsection.

Faced with the new price of movies but less income, the students will take a minute to draw the new (or “phantom”) budget line, and then decide their optimal combination of slices and movies given this new constraint. Then, we ask each student whether this new combination is still preferred, not preferred, or equally preferred to the original bundle, A. If it is still preferred, we will take away another incremental amount of income and repeat the exercise. If it is not preferred, we will add a bit more income. Any inconsistencies or violations of the assumptions of ordinal utility theory listed above are noted. Once a bundle (call it B) constructed under the new price of movies but with less income is equally preferred to the original bundle A (with the higher price of movies but more income), we have isolated the income and substitution effects and the exercise is finished, save for the accounting and completion of the diagram.

\(^4\) The exercise should be administered after students have been exposed to (1) the basic axioms of consumer theory (such as completeness, transitivity, and non-saturation), (2) the pivoting and shifting of budget lines for price and income changes, and (3) the conditions required for consumer optimization.

\(^5\) It will be helpful to select two goods that have some degree of substitutability (or to require convexity of preferences outright) to encourage an interior solution. Of course, it may be interesting to allow for corner solutions, perfect substitutes, perfect complements, quasi-linear utility, and so on.
The students will mark this bundle and the quantity of movies purchased. It is also noted that the dollar amount of income “taken away” to reach the old utility level is the (-) compensating variation. The pure substitution effect is the change in the quantity of movies from the original bundle A to bundle B (they can now sketch in indifference curve U1 so that it is tangent to both A and B), indicating their choice under the new lower price of movies but at their original utility level. The income effect is the change in the quantity of movies from bundle B to bundle C. Students will note at this point whether movies are an inferior or normal good based on the movement from B to C.

Sample Outcome

Figure 1 presents an outcome for a hypothetical student based on the data and instructions from the handout in Appendix A. Students will work out on graph paper the details of the steps outlined in that document. The setup is as follows. The student has $16 and the price of slices is fixed throughout at $1. To allow for a larger choice set, the students may be permitted to purchase fractions of goods; I allowed them to purchase a half a slice of pizza, but movies could not be divided. The student in Figure 1 chooses bundle A (8 slices, 2 movies) when the price of movies is $4 and bundle C (8 slices, 4 movies) when the price of movies is $2. She draws the diagram as well as the corresponding segment of her demand curve for movies (not shown here). The movement from A to C is labeled as the total effect. Next, $2.50 worth of income is removed, the student draws the “phantom” budget line (with the price of movies still at $2), and chooses B (7.5 slices and 3 movies) as her utility-maximizing bundle under this new constraint.

Figure 1: Sample Student Outcome

In the figure above, the substitution and income effects are A to B’ and B’ to C, respectively. A note is in order here on what the student’s options are if they are to keep the assumptions of ordinal utility intact, and instructors are likely to have to field some questions at this point. Our hypothetical student should indicate that B is strictly preferred to the original bundle, A. To see why, the student should consider a point like W, and note that B was revealed preferred to W (since B was chosen when W was affordable) and W is preferred to A (since it contains more pizza and the same number of movies). Therefore, B must

---

*This can be related back to the replacement of peaches with plums from the motivating example of the previous subsection.*
be preferred to A (by transitivity). In fact, for this particular student, the only way that A and B could be equally preferred at this point is if A contains only one movie. Instructors should be on their toes for any violations of the assumptions. Some hints are provided in the handout to keep them on track, and may be provided at the instructor’s discretion.

So, for our hypothetical student, at least one more iteration of the exercise is necessary. She will now be faced with the problem of allocating $11 of income to slices ($1) and movies ($2). At this point, she indicates that B and A are equally preferred, and thus both must lie on the original indifference curve U1. Put another way, she determines that if any more income is removed, she will be worse off than she was when the price of movies was higher. Therefore, she has isolated both the income and substitution effects at this point, and no additional income needs to be removed. The student notes that the compensating variation is $5, and that movies are a normal good for her. She sketches in U1 and U2 to complete the “textbook” diagram, and notes that the substitution effect of the price change is +1 movie, the income effect is +1 movie, and the total effect is +2 movies.

Assessing the Effectiveness and Student Perceptions of the Exercise

There are a number of ways for an instructor to assess the effectiveness of the exercise. In Appendix B, I provide a multiple choice quiz that could be administered to control and treatment groups. This ten-question quiz is based on a price increase (the scenario for the exercise in Appendix A is a price decrease) for an inferior good. The quiz assesses the students’ understanding of the salient features of the exercise: substitution, income, and total effects; the assumptions of ordinal utility theory; and the concept of compensating variation. One could perform difference in means or proportions tests on the overall quiz scores for the two groups, on individual questions, or on groups of questions dealing with related ideas. For instance, questions 1, 7, 8, and 9 test their understanding of income levels, the income effect, whether the good is normal or inferior, and compensating variation; questions 4, 5, and 6 involve the substitution effect, constant utility, and a change in the relative price.

I assessed the effectiveness of the exercise in my own course as follows. One section of intermediate microeconomics participated in the activity (N=35). These students received only the handout provided in Appendix A and some individual reinforcements during the exercise. In the other two sections of the course (N=72), I presented the material as I always have: a traditional lecture that includes a numerical example. The same amount of time (75 minutes in total) was spent in the lecture as for the class exercise, the latter of which included 15 minutes of reflection at the end. All students were informed that, in the next class meeting, a quiz would be administered to assess their understanding of the key concepts. Those in the control group were instructed to use their notes from the lecture to prepare for the quiz, and to consider cases other than what I presented in class. Similarly, students in the treatment group were to use their own outcome of the activity to prepare, and were asked to consider other possibilities as well.

I pooled the two sections receiving the lecture together to form a single control group. To explore potential selection bias, I tested for differences in pre-treatment grades of the two groups. Specifically, I found no statistical difference in their average mid-semester grades, which included two problem sets and a midterm examination that were completed prior to the exercise. Thus, it does not appear that selection bias is a large concern. Using the post-treatment quiz provided, I found that for my course, the stand-alone exercise was as effective as delivering a traditional lecture on the material, as there was no significant difference between groups in the tests discussed above. Additionally, to examine the possibility that the benefits of the exercise accrue over time, I extracted the questions on the final examination dealing with income and substitution effects, but again found no statistical difference in the scores of the two groups on these questions.

In reflecting on the exercise, students could be asked to consider how much they would be willing to pay for a Netflix subscription offering the discount, and to relate this to the compensating variation.

Students were made aware of their rights as research subjects, and only those who consented to be a part of the quiz and assessment are included. IRB approval for this project was attained (SUNY Oneonta IRB Application #2016-14), and the exercise, quiz, and survey were administered in the spring of 2016.

I assumed both equal and unequal variances in a litany of tests. The results are qualitatively similar and available upon request. Average quiz scores for the treatment and control groups were 5.1 and 5.3, respectively.
How the activity compares to traditional lecturing or other means of delivering the content is of course instructor-specific. The assessment can be done by testing the relative performance of the groups on the quiz provided. It is left to the instructor to determine how best to incorporate the activity into one’s own course (either as a substitute or complement), and to consider alternative ways of administering it. This is discussed more in the concluding section, along with some potential variations on the exercise.

Students in the treatment group gave generally positive responses to a survey administered upon completion of the exercise and quiz. The results are displayed in Table 1. On average, students indicated that they felt they had learned the concepts from the exercise, and that they would like to have more concepts illustrated using class activities.

Table 1: Student Perceptions of the Exercise (N=32)

<table>
<thead>
<tr>
<th>Survey Question</th>
<th>Ave. Score (t-stat)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The exercise helped me to understand the substitution effect of a price change.</td>
<td>2.22 (-4.13)</td>
</tr>
<tr>
<td>The exercise helped me to understand the income effect of a price change.</td>
<td>2.19 (-4.33)</td>
</tr>
<tr>
<td>The exercise helped me to understand the total effect of a price change.</td>
<td>2.16 (-4.53)</td>
</tr>
<tr>
<td>The exercise helped me to understand the concept of compensating variation.</td>
<td>2.47 (-2.52)</td>
</tr>
<tr>
<td>I would like to have more economic concepts illustrated with class exercises rather than lecturing.</td>
<td>2.34 (-3.20)</td>
</tr>
</tbody>
</table>

Note: Students were asked to rate each statement with 1=strongly agree, 2=agree, 3=neutral, 4=disagree, 5=strongly disagree. T-statistics are in parentheses (Null hypothesis: Scores are equal to 3.)

Discussion and Conclusion

This paper offers a self-contained class exercise (with minimal setup costs) for teaching the income and substitution effects of a price change using revealed preferences. In my course, the exercise was found to be as effective as lecturing. Furthermore, students enjoyed the exercise and on average indicated that they would like to have more economic concepts presented with methods other than lecturing.

For the initial run of the activity, I presented students with a specific scenario using movies and pizza – two items near and dear to college students – and I used the exercise as a strict substitute for lecturing. It could of course serve as a complement to the lecture, perhaps administered as a take-home assignment. I suspect that the activity will be most beneficial in my own course when it is incorporated this way. The exercise also provides an opportunity for students to work in groups to examine, for instance, “society’s” preferences under various social choice mechanisms.

The activity presented here allows students to discover the salient features of demand theory through their own introspective choices. In a variation using real money, students could (within funding limits) first choose the two most desirable items from a specific list, and then work with this selection throughout the exercise. The list could include items such school supplies and a variety of chocolates and candy. A variation such as this necessitates only a rescaling of the prices, after which students make their initial allocation and then carry out the exercise in exactly the same way. To ensure that all income is spent (non-saturation), the instructor would take back any unspent money at the end.

Additionally, more flexibility could be built in to the exercise by allowing for corner solutions, perfect substitutes or complements, quasi-linear utility, and so on. More advanced students may be presented with a specific utility function for which they chose the parameters. However, placing too many restrictions on the choices students are permitted to make diminishes the main purpose of the exercise, which is for them to discover how the income and substitution effects are at the core of their own demand for goods and services. As choices are restricted, the exercise becomes closer to a traditional homework assignment or example from a textbook.

References


Appendix A: Handout for (up to) a Three-Round Version of the Exercise

**RULES:**

- You have $16 for the weekend to spend on on-demand movies (X) and pizza (Y)
- The prices are $4 per movie and $1 per slice of pizza
- You may buy ½ a slice for ½ the price if desired, but you can only buy movies in whole quantities
- You have to eat something, and you have to be entertained or you’ll suffer from boredom!
- We’ll require the spending of all income, completeness, transitivity, and non-saturation
- Notation: “>” means “is strictly preferred to” and “~” means “is equally preferred to”

**STEPS:**

1. Draw the budget line (to scale and using a ruler), and mark all of the feasible combinations of slices and movies with dots.
2. Choose the feasible combination of slices and movies that maximizes your utility. Mark it as bundle A.

**Now, consider a **DECREASE** in the price of movies by 50% (that is, to $2 per movie).**

3. Draw the new budget line, and mark all of the feasible combinations of slices and movies with dots.
4. Choose the feasible combination that maximizes your utility when faced with the same amount of money income ($16) but the new, lower price of movies. Mark it as bundle C. Note that C>A.

**Now, suppose $2.50 of your income is **REMOVED**, but the price of movies remains at $2.**
5. Draw the new budget line (given that you now have $13.50 in money income), and mark all of the feasible combinations of slices and movies with dots.
6. Choose the feasible combination that maximizes your utility when faced with this lower level of income as well as the lower price of movies. Mark it as bundle B.
7. Answer this question: From your perspective, is B\(\succ\)A or is B\(~\succeq\)A?

[Hint: If bundle A consists of 1 movie, then the answer could be either B\(\succ\)A or B\(~\succeq\)A, depending on your preferences. If A contains either 2 or 3 movies, then it must be that B\(\succ\)A given our assumptions.]

- If B\(~\succeq\)A at this point, then go to step 14.
- If B\(\succ\)A, then erase “B” from your diagram and continue on.

Now, suppose an additional $2.50 of your income is removed, but the price of movies remains at $2.

8. Draw the new budget line (given that you now have $11 in money income), and mark all of the feasible combinations of slices and movies with dots.
9. Choose the feasible combination that maximizes your utility when faced with this lower level of income as well as the lower price of movies. Mark it as bundle B.
10. Answer this question: From your perspective, is B\(\succ\)A or is B\(~\succeq\)A?

[Hint: If bundle A consists of 1 or 2 movies, then the answer could be either B\(\succ\)A or B\(~\succeq\)A, depending on your preferences. If A contains 3 movies, then it must be that B\(\succ\)A given our assumptions.]

- If B\(~\succeq\)A at this point, then go to step 14.
- If B\(\succ\)A, then erase “B” again and continue on.

Now, suppose an additional $2.50 of your income is removed, but the price of movies remains at $2.

11. Draw the new budget line (given that you now have $8.50 in money income), and mark all of the feasible combinations of slices and movies with dots.
12. Choose the feasible combination that maximizes your utility when faced with this lower level of income as well as the lower price of movies. Mark it as bundle B.
13. Answer this question: From your perspective, is B\(\succ\)A or is B\(~\succeq\)A?

[Hint: At this point, B\(~\succeq\)A no matter what was in bundle A, because if another $2.50 worth of income is removed, then none of the original (pre-price-decrease) bundles would be affordable.]

14. You have isolated the income and substitution effects for a price decrease at this point! Sketch your indifference curves in, labeling them U1 (bundle A and the “final” bundle B) and U2 (bundle C) for the original and post-price-decrease utility levels, respectively.
   - Label the change in the number of movies from A to B as the “substitution effect” (no change in utility, only the price ratio),
   - Label the change from B to C, if there is one, as the “income effect” (no change in the price ratio, only income)
   - Label the change from A to C as the “total effect” (= SE + IE) and sketch the demand curve over this range

15. What is the compensating variation?

16. Are movies a normal, inferior, or neutral good for you?
Appendix B: Sample Quiz for Assessing the Effectiveness of the Exercise

Consider the following diagram showing various budget lines and bundles of goods X and Y:

Along the four budget lines shown, the consumer’s utility-maximizing bundles are D, R, K, and L. (G is not a utility-maximizing bundle.) These choices fully separate the income and substitution effects for the price change depicted below.

Suppose that initially the prices of X and Y are both $2 per unit, and the consumer chooses bundle D.

1. How much money income does the consumer have?
   a. $12
   b. $24
   c. $48
   d. None of the above

Now suppose that the price of X rises to $4 per unit.

2. Which movement represents the total effect of the price change?
   a. K to L
   b. D to L
   c. D to K
   d. K to R
3. Which of the following diagrams correctly depicts the segment of the demand curve for good X that is associated with the money income of question 1 and the price change of question 2?

a. 

b. 

c. 

d. None of the above
4. Which of the following statements is true?
   a. L is strictly preferred to D, G, K, and R
   b. D is strictly preferred to G
   c. G and R are equally preferred
   d. None of the above

5. Which of the following characterizes the substitution effect of the price change?
   a. 9 fewer units of X
   b. 7 more units of X
   c. 2 fewer units of X
   d. 2 more units of X

6. At which two points is the consumer’s utility the same?
   a. L and R
   b. G and R
   c. K and D
   d. L and D

7. Which of the following characterizes the income effect of the price change?
   a. 9 fewer units of X
   b. 7 more units of X
   c. 2 fewer units of X
   d. 2 more units of X

8. What type of good is good X?
   a. Normal
   b. Inferior
   c. Giffen
   d. Neutral

9. What is the compensating variation of the price change?
   a. $2
   b. $4
   c. $8
   d. $16

10. Generally speaking, which of the following statements accurately characterizes the income and substitution effects of a price change?
    a. The income effect represents the change in the quantity demanded that is independent of the change in relative prices
    b. The substitution effect represents the change in the quantity demand that is independent of the change in real income
    c. The substitution and income effect work in the same direction when the good is normal but in opposite directions when the good is inferior
    d. All of the above
Highly Visual, Concept-Mapped Presentations for Microeconomics with Prezi

Thomas P. Andrews¹ and Kristen Doran²

Abstract
Prezi (Prezi.com) is a web-based slideshow presentation system that allows presenters to create highly visual, engaging presentations while embedding individual concepts within the broader context using a concept map. In this paper, the authors discuss the advantages of using visually rich, concept-mapped Prezis. We also reduce startup costs by offering a series of ready-made, free presentations suitable for microeconomics that are fully animated and completely customizable. We also include a customizable concept map that allows each subject to be presented within a larger context.

Introduction
In this paper, we suggest a new approach to presentation slideshow design and recommend delivery via Prezi (Prezi.com). Prezi is a unique, web-based presentation system that makes it possible to embed ideas and media within a concept map, so that ideas can be presented within an overarching, contextual framework. Prezi also facilitates the use of images, videos, and color to create highly visual, engaging displays.

Concept Mapped Presentations with Structure and Context
In their primer for new economics instructors, Sheridan, Hoyt, and Imazeki (2014) invoke an analogy of two construction workers. The first worker is “laying bricks” while the second is “building a cathedral.” Despite the similarity of each task, the worker who is busily constructing a cathedral experiences greater job satisfaction than the brick-layer. The difference exists because the builder has valuable context that the brick-layer is missing. In the primer, the authors suggest that frequent reminders of the context and overarching themes of a presentation can similarly increase student satisfaction.

Introducing new topics to students should provide both structure and context. Returning to the cathedral example, the cathedral provides the structure. Each brick also has context based on its location relative to the other bricks. Bricks on the foundation have different context than those on a spire. To be most effective, concepts in the presentation have to be given structure, and each concept needs context. This idea is generally known as “concept mapping” (Novak 1990). A concept map is a hierarchical structure, which illustrates the relationships between concepts. Concept maps have been used effectively in a number of disciplines to increase learning effectiveness.

Horton et al. (1993) conducted a meta-analysis of research on the effectiveness of concept mapping as an instructional tool in science disciplines. Their findings noted positive effects on both student achievement and attitudes. More recently, Erdoğan’s (2016) meta-analysis using quantitative studies that compared concept mapping instructional strategies and traditional teaching methods found that concept mapping strategies led to greater academic success. In this meta-analysis, the use of concept mapping as an instructional tool generated positive benefits on academic success that included recall, problem solving,

---
¹ Professor, Department of Economics and Finance, West Chester University of Pennsylvania
West Chester, Pennsylvania, tandrews@wcupa.edu. We would like to thank two anonymous referees for their helpful comments.
² Associate Professor, Psychology, Delaware County Community College, Media, Pennsylvania, kdoran4@dccc.edu
³ Prezi can also be used off-line using the desktop version.
concept learning, developing critical thinking skills, and non-academic outcomes such as attitude (Erdoğan 2016).

While the use of concept maps is common in other disciplines, the tool has had limited exposure in economics. Marangos (2003), Marangos, and Alley (2007), Burdina (2015), and Chei-Chang (2009) present notable examples of the application of concept mapping in economics. Much like in Erdoğan (2016), Chei-Chang (2009) found that the use of concept mapping improved student achievement, attitudes, and even increased exam scores. While all four studies note the positive effects of concept mapping, each author has a different approach to the map. Marangos (2003) and Marangos and Alley (2007) use what would probably be considered a traditional concept mapping approach that emphasizes connections between terms. In this approach, each term is in a box that is connected via arrows or lines to other boxed terms. An example is shown in Figure 1.

**Figure 1. Concept map**

![Traditional concept map](image)

Other authors’ maps are augmented with text and graphs. Chei-Chang (2009) adds emphasis to the connections by adding text to each arrow, and Burdina (2015) goes further by adding explanatory text to the terms in the boxes and graphs when appropriate. Burdina calls their approach a “Chapter Concept Map.” In fact, Burdina’s maps are essentially a typical instructor’s slideshow re-configured as a concept map. Given that concept maps increase academic achievement, we propose using a concept map in a presentation using Prezi.

There is no uniform standard for constructing a concept map; it seems to be as much art as science. Figure 2 shows a concept map in Prezi for microeconomics developed by the authors. This map adopts a hierarchical spatial arrangement for the concepts. Foundational material (demand, supply, production possibilities) is at the bottom and higher-order ideas (models of market structure) are higher up. Each of the spaces in the map can be filled with appropriate concepts and more detailed information.
Figure 2. Concept map for microeconomics

Figure 3 is an example of a concept map for production possibilities, which can be nested in the bottom left box in Figure 2.

Figure 3. PPC subject map

Adding Visuals

To combat apathy and increase student engagement, Sheridan et al. suggest that lecturers provide “evocative visuals and video clips.” (2014 p.847) This rather minor modification can make a world of difference in a presentation. Smith and Woody (2000) reported that multimedia-teaching strategies have benefits, especially for learners that are visually oriented. Further, Johnson, and Christensen (2011) investigated the impact of using presentations with frequent visuals and minimal text and found that students reported greater satisfaction. Implementing the use of visually enriched presentations, combined
with the benefits of concept mapping, is one way to respond to the increasing focus on student success, satisfaction, and retention.

Consider, for example, the two slides presented side-by-side in Figure 4. The slide on the left is a typical publisher’s slide with three features of perfect competition. The slide on the right was created with a focus on images rather than text. In the “many sellers” bullet on the left, “many” means that no single firm, by changing its output can make a noticeable difference in price of the product. The image of candles serves in place of the “many sellers” bullet on the left. The image is designed to reinforce the concept of “many,” by virtue of analogy to a term coined by Ernst Weber from psychophysics, a subfield of experimental psychology. Fanche & Alexandra describe Ernst Weber’s term of a “just noticeable difference” (JND) as “the minimum amount of difference between two stimulus intensities necessary for an observer to tell them apart” (2012 pp. 167-171). In this analogy, the JND is the single candle and any number of candles more than 50 would satisfy the concept of “many.” The candle image along with the story is more effective and engaging than a single bullet helping students store this concept in their memory. But even if the recall is not noticeably enhanced, the image of the candles is more engaging than the corresponding bullet with text. The use of visuals, combined with “many sellers” contained within market structure on the concept map, will further enhance student learning.

Figure 4. Text versus images

Ideally a presentation can be both visually engaging and concept mapped. Prezi is the ideal presentation delivery system for just such a presentation. Further, the use of visually enriched presentations within a concept map moves away from the traditional delivery method of content where a teacher explains a bullet point while students write. The dynamic delivery method explained here replaces this method with teacher explains concepts through story telling, creative imagery, and context and students pay attention and understand.

Prezi

Unlike traditional slideshow presentations, which follow a linear progression, Prezi uses a canvas. Like a painting, concepts are laid out in two (or three) dimensional space. Rather than advancing in a linear sequence through slides, the viewer navigates the Prezi using what Prezi calls the Zooming User Interface (ZUI). With each click, the viewer’s attention is focused on a portion of the canvas. In this way, it is possible to display all of the concepts on the canvas to the audience at once allowing the viewer to see each concept (brick) in the context of a larger structure (the cathedral). The viewer’s attention is drawn to one concept or group of concepts at a time by zooming into the “slide” that represents those concepts. Zooming out on the canvas easily restores the structure and context.

Figure 5 shows a screenshot of a visually-rich, concept-mapped Prezi presentation for Production Possibilities Curves. When played, the Prezi will advance through the subject material shown in Figure 3 but within the greater context and structure provided by the map shown in Figure 2. In addition, the presentation has embedded images and videos to further engage the audience and keep their attention.
The Prezi canvas has certain advantages in adding images and videos. First, all images can be placed, moved, cropped, and resized without restrictions within Prezi. Second, users can source images from online within Prezi by using the built-in image search engine. YouTube videos are seamlessly embedded and playback full-screen without introductory advertising by simply copying the link into the Prezi canvas. The most crucial benefit to presenters is that Prezis are easily shared via a hyperlink and is instantly updated when edited or updated without having to update the hyperlink.

**Getting Started with Prezi**

To begin, users must sign up for a Prezi account. Prezi operates on the “freemium” model where more advanced services come at a cost. Basic accounts are free, as are feature-rich accounts for students and educators (with an “.edu” email address). There are also a number of high-quality tutorials available on the site. Because Prezi encourages open access and sharing, once logged in, users have access to thousands of pre-existing Prezis and templates including those described in this paper.

To better understand Prezi, it is best to start with the basic elements of a Prezi. These include the canvas, frames, and content. The canvas is the entire area where the Prezi is displayed (the space occupied by the cathedral). Frames are shaped like brackets, circles, or rectangles that carve out smaller areas of the canvas where the viewer’s attention can be focused (the bricks) using the ZUI. By design, this feature makes the system ideally suited for concept mapping. Finally, the content can include text, images, videos, plus an array of lines, arrows, and shapes are available to populate on the canvas and convey meaning, structure, and context.

The time it takes to adopt a new system is a significant barrier. To reduce this barrier, the authors have developed a series of Prezis that form a “common core” of microeconomics topics: production possibilities, markets, production and cost, and so forth. Links to these Prezis are provided in the Appendix. Within each Prezi, content is grouped in frames and each frame functions much the same way as a traditional slide. The Prezis are already animated, so that material fades in sequentially. The Prezi modules have few images and no videos, or color. Like a new canvas, they are intentionally bland so that the presenter can choose their preferred colors and materials. Finally, there is a single “concept map template” that illustrates the structure and context in two-dimensional space as one suggestion for organizing the content. The concept map serves as the background (the cathedral) for the topic Prezis (the bricks).

The concept map template provided in the Appendix leverages the two-dimensional canvas by including frames for each of the four most commonly taught product market structures along with the foundational material that is used in each market structure (Figure 2). The brackets and circles are frames that contain content for a particular topic. The market structure (bracket) frame contains four separate circle frames. The three bracket frames below are foundational topics that will be used in each frame in the market structures. When the material is presented, the viewer’s focus is narrowed to a targeted area. By
utilizing this presentation platform and this organization style, the presenter can easily illustrate detailed concepts while also providing a visual for the overarching themes to the audience.

In theory, it would be possible to add enough frames and content to cover all the material in an entire course in a single Prezi. In practice, though, if too much information is entered in a single presentation, the Prezi may not function well. To counter this issue, a presenter must organize material into shorter Prezis around a single topic, while designing them all to fit in a single uniform concept map. As an example, Figure 3 illustrates the Production Possibilities Curve (PPC) Prezi.

The PPC Prezi contains six sub-frames (the equivalent of six slides). Each frame highlights a subtopic for a presentation on production possibilities. As the viewer clicks through the Prezi, the system shifts focus to each frame. In addition, content is added sequentially within each frame as the Prezi is advanced. Each frame is constructed with a minimum of text and the last frame is left empty. This is a good place to insert one or more videos or illustrative examples. All Prezis in the Appendix are freely available and can be copied and modified without restriction.

**Combining Subject Templates with the Base Template**

The Prezis that focus on a specific subject can be used as is, but alone, they lack context and structure. Applying a concept map using a three-step process provides the structure. After a presenter establishes a Prezi account, they need to click on the links provided in the Appendix, and use the “save a copy” button to make their own copies of the concept map template and the subject Prezis. These Prezis will now appear in the presenter’s account with the title of “copy of …”. They are now the presenter’s personal copies and can be renamed, edited, revised, and made private to incorporate preferred colors, images, and videos.

The next step is to add a concept map template to the subject modules. The best way to do this is to edit the concept map then capture a screenshot of the concept map and save it as a .pdf file. This file then serves as the concept map, and it can be added to each subject module as an image and can be used as a student handout. From there, the entire subject module can be positioned properly on the concept map image (the .pdf image).

Finally, after the concept map and subject modules are combined, the presenter can add images, animations, and videos to enhance the presentation and set a new path through the content frames. The first row in the table in the Appendix has a link to a brief “How-to” video. Again, Figure 5 illustrates a colorful, fully animated, image, and video enhanced PPC module embedded in the concept map.

**Student Feedback**

It’s hard to justify a significant change in a course without some evidence of effectiveness. The research on the use of concept mapping and visuals has shown positive benefits to learning and student satisfaction. Prezi can be used to blend these two teaching strategies. While the authors have not conducted a scientific test of effectiveness of this approach, we have surveyed student opinion. In the past year, students in the primary author’s 100 level microeconomics class were asked to provide feedback about course materials including questions about the textbook, the online homework system, other course supplements, and the Prezis. The anonymous “mid semester course material improvement survey” was delivered by email to 4 sections (3 sections with approximately 35 students each and one section with approximately 90 students). The survey contained questions about the “usefulness” and the “importance” of the Prezis. The concept-mapped, visually enhanced Prezis scored high in terms of helpfulness. Approximately 35% of the class gave the highest rating for helpfulness and 65% gave one of the two highest ratings. In addition, students were asked to rank the importance of all the course materials from 1 (most important, definitely keep it) to 7 (least important, definitely get rid of it). As seen in Table 1, the Prezis got more first place votes (101) than any other resource utilized in the course. In comparison, the Openstax textbook got the fewest first-place votes, and the online homework system (Sapling Learning), which was the most expensive portion of the course, received just 10 percent of the first place votes. While it might not be surprising to find that the presentation materials were so highly valued by the students given the ubiquity of slideshow presentations in the discipline, it is important to consider that since the slides contain very little text, their value likely lies almost entirely in the images and videos they contain.
Table 1. Prezi Ratings

<table>
<thead>
<tr>
<th>Course material</th>
<th>First Place Votes for “Most Important” Class Resource (n=246)</th>
<th>Percent of first place votes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Prezis</td>
<td>101</td>
<td>30.0</td>
</tr>
<tr>
<td>Collaborative learning exercises</td>
<td>77</td>
<td>22.8</td>
</tr>
<tr>
<td>Audience response system</td>
<td>61</td>
<td>18.1</td>
</tr>
<tr>
<td>Instructor videos</td>
<td>47</td>
<td>13.9</td>
</tr>
<tr>
<td>Electronic homework</td>
<td>34</td>
<td>10.1</td>
</tr>
<tr>
<td>Openstax Textbook</td>
<td>17</td>
<td>5.0</td>
</tr>
</tbody>
</table>

There was also an opportunity in the survey to leave an open-ended comment about the Prezis. These comments were far more positive than negative. A representative sample of these comments is shown in Table 2. The positive comments seem to support the use of Prezis, while for the most part, the negative comments relate more to the pace of delivery.

Table 2. Student Comments or Suggestions Regarding the Prezis

- The Prezis were very useful, especially being able to go back and look at them when not in class.
- I enjoy the Prezis in class because they were more interactive to me over powerpoints. It helped me focus by basically breaking the lecture down into different parts.
- I thought they were incredibly helpful. They provide the student with a backup resource while working through the homework, sapling and while studying for the exam.
- Great resource for us students to use if we need help understanding material
- put a little more material/explanations in it so so when going back to study its a little more helpful
- like them a lot and that they were available online
- The Prezis were much more entertaining than powerpoints and they kept me engaged and involved.
- They're really good but just during class slow down!
- good set up. not real the biggest fan of prezi since it jumps around in random order, but it is still good.
- Helpful and the presentation is nice. Just confusing sometimes when it jumps from point to point
- i liked the prezis
- Keep using Prezi!
- They are helpful to look back on before taking the online quiz's and also for Written Homeoworks
- Try not to overcrowd them
- Go more in depth with explanations; limit ambiguity
- Prezis were helpful at time although I found it hard to navigate around the prezis at certain times.
- go slower and/or have a printable version to follow along in class. trying to keep up and copy everything while trying to listen and comprehend all at once is very hard when you go through them so fast; and i look over them before class. also, add a clearer how-to for how to craft graphs and the method of filling in graph information

Conclusion

In this article, we have reviewed the advantages of using Prezi presentations using an image-laden concept map. Concept maps, images, and videos are all useful in maintaining student’s attention and improving student satisfaction. In addition, there is evidence that these elements will improve student learning. The Prezi platform allows for a synergistic combination of approaches. Each portion of the visually rich presentation takes place with appropriate structure and context. This approach is supported by positive student feedback. Finally, we offer a suite of fully animated, completely customizable, concept mapped Prezis that will help presenters deliver dynamic and engaging presentations.
References


## Appendix

### Prezis for Microeconomics

<table>
<thead>
<tr>
<th>Topic</th>
<th>Hyperlink</th>
</tr>
</thead>
<tbody>
<tr>
<td>Video Tutorial</td>
<td><a href="https://drive.google.com/file/d/0B3ouTHhS-oCuRlc1cnIlRy02MDA/view?usp=sharing">https://drive.google.com/file/d/0B3ouTHhS-oCuRlc1cnIlRy02MDA/view?usp=sharing</a></td>
</tr>
<tr>
<td>Concept Map Template</td>
<td><a href="http://prezi.com/-0ph7wcl4ckc/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/-0ph7wcl4ckc/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Production Possibilities</td>
<td><a href="http://prezi.com/l1phqojseaf/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/l1phqojseaf/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Demand</td>
<td><a href="http://prezi.com/bsjwmsr4bjg/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/bsjwmsr4bjg/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Supply</td>
<td><a href="http://prezi.com/i1ezc5lan600/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/i1ezc5lan600/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Supply and Demand</td>
<td><a href="http://prezi.com/pezxolyveweh/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/pezxolyveweh/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>International Trade</td>
<td><a href="http://prezi.com/7frwhspqpesq/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/7frwhspqpesq/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Price Elasticity of Demand</td>
<td><a href="http://prezi.com/nctweuxyxs/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/nctweuxyxs/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Consumer Surplus and Deadweight loss</td>
<td><a href="http://prezi.com/zqzyys8pwa/vi/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/zqzyys8pwa/vi/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Production and Cost</td>
<td><a href="http://prezi.com/qtefevbh6s3w/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/qtefevbh6s3w/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Perfect competition</td>
<td><a href="http://prezi.com/b48xtzezeigu/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/b48xtzezeigu/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Monopoly</td>
<td><a href="http://prezi.com/rcufmo9s5vnp/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/rcufmo9s5vnp/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Monopolistic Competition</td>
<td><a href="http://prezi.com/uyv5ch5kqhzf/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/uyv5ch5kqhzf/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
<tr>
<td>Oligopoly</td>
<td><a href="http://prezi.com/elrxdsuqa9u/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share">http://prezi.com/elrxdsuqa9u/?utm_campaign=share&amp;utm_medium=copy&amp;rc=ex0share</a></td>
</tr>
</tbody>
</table>
Cryptocurrencies in a Money and Banking Course

Jennifer W. Kelber¹ and Meghan Hennessy Mihal²

Abstract

Money and Banking courses study money within a limited framework: money is defined, functions of money are explained, and monetary aggregates are classified. With the advent of cryptocurrencies and blockchain technology, money and banking are in the process of being revolutionized. We provide readers with an overview of cryptocurrencies and then present an effective method for incorporating cryptocurrencies in a Money and Banking course. Specifically, we highlight three occasions where a discussion of cryptocurrencies is appropriate: the introduction of money, the structure of the banking industry, and the potential role of the central bank.

Introduction

Cryptocurrencies like bitcoin have the potential to drastically change how we think about money.³ As Böhme et al. (2015) explain, “Bitcoin is of interest to economists as a virtual currency with potential to disrupt existing payment systems and perhaps even monetary systems” (pp. 214). Likewise, blockchain technology—which was introduced with the Bitcoin protocol but is also employed in other applications—has piqued the interest of investors, regulators, and policy makers. Indeed, Tapscott and Tapscott (2016) claim “blockchain technology promises to make the next decade one of great upheaval and dislocation but also immense opportunity for those who seize it” (pp. 58). And Brainard (2016) notes “the Federal Reserve Board has established a multi-disciplinary working group that is engaged in a 360-degree analysis of fintech innovation” (pp. 1).⁴ Given the widespread interest in cryptocurrencies, it is imperative that students consider the implications of such innovations for the financial system. We contend that these issues should be addressed in Money and Banking courses.

In what follows, we offer a brief primer on cryptocurrencies and blockchain technology, with special attention paid to Bitcoin. We then put forward a method for including this material in a standard Money and Banking course.⁵ Our focus is on inquiry-based learning techniques with clearly identified goals and suggested discussion questions. After discussing our approach, we review results of student surveys to gauge effectiveness and the extent to which students value such additions. Finally, we offer some concluding remarks that reflect of our experience.

Cryptocurrencies: Explained

In the broadest sense, digital currency refers to any currency that is solely stored in electronic form. Recent developments have created new classifications for digital currency; we focus specifically on

¹ Assistant Professor of Economics, Saint Anselm College, Department of Economics and Business, 100 Saint Anselm Drive #1755, Manchester, NH 03102, email: jkelber@anselm.edu
² Associate Professor of Economics, St. Thomas Aquinas College, School of Business, 125 Route 340, Sparkill, NY 10976, email: mmihal@stac.edu
³ We are following convention in the literature by using Bitcoin with a capital “B” to denote the system, and bitcoin with a lowercase “b” for the unit of account.
⁵ The authors used Mishkin’s 11th edition of Economics of Money, Banking, and Financial Markets (2016). The additions put forward in our paper are appropriate to use in conjunction with any of the mainstream textbooks.
cryptocurrencies. Dwyer (2014) explains, “Bitcoin and similar digital currencies are called cryptocurrencies by some because the underlying algorithms and security are intimately related to digital cryptographic algorithms. A publicly available database records every trade of currency. Every bitcoin is associated with an address and a transaction is a trade of bitcoins from one address to another. This database is called the ‘blockchain’” (pp. 83). It is the manner in which transactions are conducted with Bitcoin that makes it a cryptocurrency. As Böhme et al. (2015) indicate, “Bitcoin relies on two fundamental technologies from cryptography: public-private key cryptography to store and spend money; and cryptographic validation of transactions” (pp. 216). Currently there are 640 cryptocurrencies listed on CoinMarketCap.com where the U.S. Dollar price is tracked and total market capitalization is calculated by the price per unit multiplied by number of units outstanding (White 2016). Bitcoin is by far the largest cryptocurrency with a price of $709 and a market capitalization of over $11.2 billion accounting for approximately 83.2% of the cryptocurrency market. Non-Bitcoin cryptocurrencies are known as “altcoins” and account for approximately 16.8% of the market for cryptocurrencies with a total market capitalization of approximately $2.3 billion. Böhme et al. further explain that Bitcoin is able to establish scarcity of the money supply through an incentive rewarding of bitcoin to the participants of their self-regulation bookkeeping, which will be explained in further detail in the next section. Due to the lack of central authority inherent to not just Bitcoin, but to cryptocurrencies in general, there exists a greater degree of anonymity for the users than traditional currencies or a more general digital currency.

Bitcoin successfully integrated cryptography with the currency itself thus developing the blockchain protocol, which many argue is the major contribution of Bitcoin. The innovation of the blockchain protocol solved the classic double-spending problem without the use of a central authority. The double-spending problem arises when a digital currency is spent or transferred more than once. Bitcoin enables one party to send money electronically directly to another party without the use of a third party. It accomplishes this through the use of the blockchain ledger. By broadcasting every transaction to the entire network and requiring verification by multiple miners, Bitcoin is able to maintain authenticity. Luther (2016c) notes that Bitcoin experiences the first-mover advantage in developing the blockchain protocol. As altcoins have developed, they have followed in Bitcoin’s footsteps incorporating the blockchain protocol technology and they “might make use of a second-mover advantage to outcompete bitcoin in the long run,” (Luther 2016, pp. 399). Thus, altcoins have the ability to capitalize on Bitcoin’s positive contributions, and at the same time realize the shortcomings of Bitcoin and adjust accordingly. Regardless of whether or not Bitcoin’s prominence via its first-mover advantage continues, it is clear that the blockchain protocol will last.

**Bitcoin: The Details**

Created in 2008 by an anonymous programmer known as Satoshi Nakamoto, Bitcoin is a payment protocol that enables the transfer of ownership of the cryptocurrency called bitcoin. In introducing the cryptocurrency, Nakamoto (2008) states “a purely peer-to-peer version of electronic cash would allow online payments to be sent directly from one party to another without going through a financial institution” (pp. 1). As noted above, other cryptocurrencies exist. However, given Bitcoin’s widespread use, its large market capitalization, and its status as the first mover in the market for cryptocurrencies, we focus our attention on Bitcoin. Luther (2016c) supports this, stating “If one is interested in switching to a cryptocurrency, bitcoin is the obvious choice” (pp. 399). Though only a small fraction of worldwide transactions take place via bitcoin each day, the number of daily transactions has increased steadily since its inception. From January 1, 2010 to November 1, 2016, the number of daily bitcoin transactions increased from 119 to 268,418 (Blockchain 2016). Additionally, it should be noted that the value of a bitcoin fluctuates and has been rather volatile since its inception. For example, the value of one bitcoin surged above $1100 in 2013 and sank below $200 in 2015. In the time since, it has stabilized somewhat (Coindesk 2016). It is apparent that not only is the daily volume trend on an upward trajectory, but that Bitcoin’s protocol is already shifting how we think about money. Economists, central banks, financial intermediaries, and individuals are taking notice of Bitcoin.

---

How It Works

Owners of bitcoin store their funds in a digital wallet such as Coinbase. Each owner has both a public “key” and a private “key.” Only the public key is known to the entire public network, but both keys are used to conduct a transaction. If a buyer purchases a good from a seller for the price of 1 bitcoin this transaction is broadcast to the entire public network using the public keys. The transaction is kept safe and pseudonymous via the use of private keys, similar to a signature on a paper check. As Böhme et al. (2015) explain, about every ten minutes, new transactions are grouped together in a “block” of recent transactions. To ensure transactions have been authorized, new blocks are compared and then yield a link of blocks, referred to as the “blockchain.” Each transaction is then verified by miners who must solve a computationally difficult, but easy to verify, problem based on the pre-existing contents of the block. The miner who solves the problem is awarded with a fixed number of newly minted bitcoin (Velde 2013). Since each transaction is broadcasted to the entire network, this global network of interconnected individuals actually takes the place of financial intermediaries (Brito and Castillo 2013). This describes the blockchain protocol that is garnering such attention and praise.

The design of the Bitcoin network is unique. Brito and Castillo (2013) compare the mining of bitcoin to the extraction of commodities. As miners verify transactions, they are awarded bitcoin, hence increasing the supply of the cryptocurrency. As time goes on the mathematical problems that miners must solve to complete verification become increasingly difficult, thus increasing the supply of bitcoin at a slower rate. Further, the amount of bitcoin rewarded to the successful miner is programmed to decrease over time, halving every four years (Velde 2013). Bitcoin’s approach to how it increases its supply is starkly different than the manner by which the money supply changes when controlled by a central bank. Böhme et al. (2015) note, “Bitcoin’s design embodies a basic version of monetary policy that does not consider the state of the real economy” (pp. 234).

Bitcoin: The Currency of the Future?

One of the most widely touted advantages is the lower transaction costs associated with bitcoin. Brito and Castillo (2013) compare the 9.05% average fee for using a wire service to the 1% average fee for completing this same transfer using bitcoin, which could potentially impact the volume of international funds transfers. This benefits both consumers and merchants, as this will lower costs on both sides of the transaction. Further, the low transaction costs could theoretically lead to a greater use by the world population, therefore giving those in economically challenged areas access to the world economy. As Andreesen (2014) states, “Bitcoin, as a global payment system anyone can use from anywhere at any time, can be a powerful catalyst to extend the benefits of the modern economic system to virtually everyone on the planet.” Additionally, in areas where hyperinflation is apparent from mismanaged currencies by governments, cryptocurrencies can provide the necessary stability for the local economy (Luther 2016c).

A second advantage of Bitcoin is its decentralized and transparent structure. As discussed previously, Nakamoto designed the Bitcoin network to be an open-source network for which there is no centralized, controlling authority. With this design, proponents argue that the currency is less prone to corruption. Though the system itself is transparent, each user of bitcoin is pseudonymous. Identities are not disclosed to one another, or to a third party, as each transaction is recorded in the public ledger via the use of public keys.

Users also have control of their transactions in the sense that unauthorized transactions cannot take place without access to the user’s private key. Luther (2016a) further highlights the fact that bitcoin transactions are effectively irreversible, so vendors should have more confidence in accepting bitcoin for transactions in a similar fashion that vendors often prefer cash. Lastly, and arguably most importantly, Bitcoin’s blockchain represents a massive technological advancement, which is already leading to further innovations.

---

7 The transactions are recorded in a recursive nature. The buyer’s transaction to the seller will also include information about where the buyer received their bitcoin (Böhme et al. 2015).

8 Buyers need to have caution because, when they conduct transactions in cash, there is no third party to step in if the goods purchased are not provided to the buyer.
Luther (2016c) argues that the incumbent money provides the largest obstacle for the universal adoption of Bitcoin. Luther (2016a) explains the network effects and switching costs associated with cryptocurrencies where switching costs are costs incurred from changing from an incumbent money to bitcoin. Luther (2016c) notes that switching costs include “the need to retool vending and automatic teller machines; to update menus and transaction records; and even to learn to calculate in terms of a new unit of account….it [bitcoin] must be sufficiently better to warrant the cost of switching” (pp. 398). Luther continues “Network effects result when the value of a good or service depends on the total number of those using it. Monies are characterised by network effects because a medium of exchange is only useful to the extent that one’s trading partners are willing to accept it” (pp. 398). The network effects are significant and switching costs are high for bitcoin, thus the conversion to the sole currency of Bitcoin is unlikely.

Since Bitcoin has no regulatory authority, Nakamoto set the supply of Bitcoin to be inelastic and predetermined to increase at a decreasing rate over time (Dwyer 2015, pp. 83). Selgin (2015) notes that “Bitcoin is far from being an ideal monetary medium from a macroeconomic perspective” (pp. 98). He further explains “Although quantity of Bitcoins will continue to increase until 2040, in that year, Bitcoin will just become another example, albeit a digital one, of a synthetic commodity money with a supply that is at best absolutely constant and, at worst, slowly declining owing to attrition” (pp. 98). Additionally, some may argue that the decentralized structure of Bitcoin could be a potential downfall because there is no government oversight. As noted by Lo and Wang (2014): “Research in monetary economics has shown quite convincingly that it is generally preferable to rely on a central bank to adjust the money supply according to economic conditions instead of relying on a money supply that fluctuates exogenously” (pp. 11). In contrast, Selgin et al. (2012) examine the performance of the Federal Reserve in regard to monetary policy and conclude that the institution has failed on several accounts including the maintenance of a stable price level and output growth. As such, Selgin et al. likely disagree with Lo and Wang in that the Federal Reserve should not have the authority to adjust the money supply as they state “the Federal Reserve system, as presently constituted, is no more worthy of being regarded as the last word in monetary management than the National Currency System it replaced almost a century ago” (pp. 592). The consequence of Bitcoin’s inelastic supply and definitive end-date for monetary expansion are two-fold: an increased likelihood of deflation and potential instability as the payments network expands (Selgin 2015). This, in combination with the lack of a monetary authority, represent potential setbacks for Bitcoin.

As previously noted, money serves three functions: it is a medium of exchange, a unit of account, and a store of value. Bitcoin serves the function of medium of exchange and the unit of account fairly well. Since its inception, bitcoin has experienced volatility in its price, rendering it a somewhat poor store of value. Dwyer (2012) compares the standard deviation of gold to the standard deviation of Bitcoin from 2010 to 2014 and found the standard deviation of gold to be significantly lower than that of Bitcoin. Dwyer’s results display a maximum standard deviation of 2.2 percent for gold whereas Bitcoin displayed a maximum standard deviation of 17 percent. However, the Bitcoin Volatility Index9 indicates that the volatility of Bitcoin is trending down. The current 30-day volatility estimate of 2.22 percent is markedly lower than it has been in past years. Further, the volatility risk may be largely negated by the consumer and the merchant if the bitcoins are to be immediately converted into fiat money. This conversion then transfers the volatility risk to the third-party intermediary.

Furthermore, the second-party storage and exchange of bitcoin may be susceptible to hacking, as evidenced by the collapse of the Mt. Gox bitcoin exchange in 2014.10 Due to its pseudonymous nature, bitcoin shares some of the disadvantages of cash: the difficulty of tracing transactions means that if the money is lost it is irretrievable. It also means that bitcoin may be used for illicit purposes. The now-shuttered Silk Road online marketplace is widely cited as illustrating Bitcoin’s illicit uses. Silk Road functioned as an online marketplace whereby buyers and sellers could interact to trade goods anonymously. Owing to the anonymity inherent to the site and the lack of restrictions on the types of goods sold, the site in part became an exchange for drugs and other illegal goods. Silk Road provided anonymity not only in communication, but also in payment as the currency used was bitcoin (Christin 2012). However, Meiklejohn et al. (2013) illustrate that the potential for Bitcoin to be used entirely anonymously is limited. The authors tracked various thefts of

---

10 Mt. Gox, the once-dominant Tokyo-based Bitcoin exchange, lost 850,000 bitcoins in February 2014 due to a supposed hacking; however, speculation has been made that it was an inside job. Mt. Gox filed for bankruptcy promptly after the loss. See Sidel et al. (2015) for more information.
bitcoin and claim, “With these thefts, our ability to track the stolen money provides evidence that even the most motivated Bitcoin users (i.e., criminals) are engaging in idioms of use that allow us to erode their anonymity...Bitcoin does not provide a particularly easy or effective way to transact large volumes of illicitly-obtained money” (pp. 12). Luther (2016c) notes that most illicit markets operating online today rely on cryptocurrencies.

Impact on the Banking System

Much of the popular debate in the media surrounding bitcoin tends to focus on whether or not it is a currency and whether or not it is here to stay. Instead, we suggest that the focus be on the possibilities and implications for the economy that arise through its use. In this section we consider the potential impacts on the banking system arising from the adoption of both Bitcoin and blockchain technology. Luther (2016c) states “the blockchain technology will be widely adopted to process digital payments….to the extent that the blockchain technology lowers transaction costs, it will likely be adopted to process digital payments” (pp. 401).

One of the advantages noted above is the potential for innovations to be built off of the Bitcoin protocol, the blockchain. As Brainard (2016) indicates, “Current developments in the digitization of finance are important and deserving of serious and sustained engagement on the part of policymakers and regulators” (pp. 1). We argue that whether or not individuals actually use bitcoin is somewhat irrelevant. What is important to understand, though, is that financial institutions are likely to utilize the technology in some form. This spells changes for the industry. NASDAQ Private Market Platform has already taken this step and has committed to use a blockchain-style digital-ledger to manage equities (Orcutt 2015). Financial costs are likely to decrease for individuals. The adoption of this technology by financial institutions will lead to drastic changes in the roles of financial intermediaries. Potential changes include those that may take place in trading, clearing, and settlement (Brainard 2016). These changes have the potential to lead to an increase in efficiencies and reduction in risks. Structural job shifts are also likely to occur; some jobs will be rendered obsolete while others find themselves in demand. With this financial innovation, banks will experience a decrease in income from transactions. They will need to establish new revenue streams, which could come in the form of unique services that a fully independent financial entity, like Bitcoin, cannot provide. Product differentiation could allow them to recapture the market. Thus, it is imperative that banks take advantage of these revolutionary changes instead of becoming consumed by the changes. Bitcoin and the blockchain represent opportunities for banks, but banks need to understand the potential and must be prepared for a shifting landscape.

While discussing the impacts on the banking sector, it is appropriate to address the potential impacts on the monetary system. Hendrickson et al. (2015) explain: “bitcoin might facilitate illicit transactions and disrupt government activities like conducting monetary policy and raising revenues” (pp. 937). As more residents in a nation adopt Bitcoin as a currency, the central bank’s transmission mechanism becomes weaker, and the nation's ability to create seigniorage decreases.

Incorporation into a Money and Banking Course

Pedagogically, there are three junctures at which it is useful to discuss cryptocurrencies, using Bitcoin as our example, in a Money and Banking course. By structuring the course this way, students come to understand the material well without feeling overwhelmed. It also makes it easy for instructors to include the material in existing courses. We provide a list of inquiry-based goals and discussion questions for each juncture. We find that these questions—and the open-ended discussions they prompt—engage students and elevate the energy in the classroom. They also help students’ understanding of the material. Table 1 illustrates our approach. Table 2 provides instructors with our suggestions for the manner in which outside sources may be effectively assigned and implemented in order to achieve the inquiry-based learning goals.
Table 1: Methodology

<table>
<thead>
<tr>
<th>Breakdown of Digital Currencies Material</th>
<th>Inquiry-Based Goals</th>
<th>Inquiry-Based Discussion Questions</th>
</tr>
</thead>
</table>
| **Part I: Introduction to Money**       | To more fully understand the functions and nature of both traditional monies and cryptocurrencies | • What is money?  
• What are cryptocurrencies?  
• How does the Bitcoin protocol, the blockchain, work?  
• Is bitcoin money?  
• What may be the future of bitcoin? |
| **Part II: The Structure of the Banking Industry** | To consider the potential implications of cryptocurrencies and blockchain technology on the banking industry | • How might cryptocurrencies and blockchain technology impact the banking industry?  
• How might banks innovate to be successful?  
• Is there more regulation ahead for Bitcoin?  
• What may be the role of the central bank in regulating such a cryptocurrency? |
| **Part III: Regulation and Role of the Federal Reserve** | To consider the ways in which the central bank may be impacted by widespread adoption of cryptocurrencies and blockchain technology | • What would be the implications of central banks issuing their own digital currencies?  
• Is an entirely digital currency feasible?  
• How could the central bank conduct monetary policy in such a regime? |

Table 2: Implementation

<table>
<thead>
<tr>
<th>Breakdown of Cryptocurrencies Material</th>
<th>Additional Scholarly Sources</th>
</tr>
</thead>
</table>
| **Part I: Introduction to Money**       | • Radford 1945  
• Velde 2013  
• Lo and Wang 2014 |
| **Part II: Implications for the Banking Industry** | • Böhme et al. 2015  
• Luther 2016b |
| **Part III: Impactful Changes for the Central Bank** | • Raskin and Yermack 2016  
• Bank of International Settlements 2015 |

The manner in which we suggest this material be incorporated into a given Money and Banking class is such that it blends organically with the existing course. The recommended readings may be assigned as external homework assignments and then discussed during class time. Naturally, we face trade-offs whenever we spend time on one activity versus another, but we contend that this time is well-spent. An ever-changing economic environment requires that courses constantly adapt to these changes.

**Part I: Introduction to Money**

Money is not always what money seems. To encourage students to think more deeply about the questions we pose, we begin our discussion of money with a review of the history of money. This is accomplished with Radford’s (1945) classic article, “The Economic Organisation of a P.O.W. Camp.” This article is extraordinarily useful in conveying to students that it need not be only dollars and cents that serve as what they generally think of as money. Radford provides a detailed illustration of the development of
money in the form of cigarettes in a World War II Prisoner of War camp. Though we travel back to 1945 to explore this development, the article broadens students’ horizons by changing their view of what actually constitutes money. Additionally, it provides students a good starting point to understand our study of money. With this expanded thinking, students are better able to comprehend cryptocurrencies.

We proceed by introducing cryptocurrencies through the use of an approachable, yet high-quality source, such as “Bitcoin: a Primer” by the Chicago Federal Reserve (Velde 2013). Velde explains Bitcoin in a manner in which students can comprehend. First, he describes the cryptocurrency. Then, he explains how the blockchain protocol works. After reading the work, students should have a clear understanding of bitcoin and be well on their way toward tracing the potential implications for the financial system.

A useful article in helping students to consider whether or not bitcoin fulfills the three functions of money is the policy perspective published as part of the Federal Reserve Bank of Boston’s Current Policy Perspective, “Bitcoin as Money?” (Lo and Wang 2014). As referenced previously, Lo and Wang provide readers with a detailed analysis of bitcoin in light of the three functions of money: medium of exchange, unit of account, and store of value. In examining the characteristics of the cryptocurrency within the framework of the traditional functions of money, students are better able to discern the nature of currency and its potential future.

At this stage in the course, students will have attained a fair understanding of what constitutes money as well as its common functions. Focusing on the inquiry-based goals and discussion questions while utilizing the three aforementioned sources will lead to robust discussion and clarity of understanding. Through the expanded exploration of cryptocurrencies, students gain a deeper grasp of money.

Part II: Implications for the Banking Industry

Most Money and Banking courses explore the structure of the banking industry. We suggest a more robust understanding would result from an examination of the potential implications on the industry arising from the widespread adoption of cryptocurrencies. After all, many students studying business, economics, and finance will go on to work in financial services. With the use of cryptocurrencies and financial institutions’ interest in blockchain technology on the rise, it is advantageous for these students to consider potential changes in the industry. As Hendrickson et al. (2016) note, “It [bitcoin] might also preclude a government from accomplishing its monetary policy goals or raising revenues” (pp. 929).

To facilitate the introduction of this material into the course we suggest the use of “Bitcoin: Economics, Technology, and Governance” by Böhme, Christin, Edelman, and Moore (2015). This quality source provides a detailed explanation of the cryptocurrency, the blockchain protocol, the uses, the risks, and the potential for regulation. The paper remains approachable without sacrificing depth, making it an ideal source for undergraduates.

Böhme et al. (2015) explain that Bitcoin lacks governance, which provides opportunity for regulation. Crime and consumer protection are given as justifications for regulatory oversight. The Silk Road incident of 2013 is used to illustrate the potential role that regulators may play and provides an opportunity to discuss the ethical obligations of regulators.11 As one of the central issues surrounding Bitcoin is the degree to which it should be regulated, we suggest additional discussion on the topic may be warranted. Luther (2016b) effectively considers the marginal benefits and marginal costs of regulation in his “Regulating Bitcoin: On What Grounds?” Luther reviews the benefits and costs in relation to three overriding concerns: consumer protection, illicit transactions, and macroeconomic policy, concluding that though benefits of regulation exist, they are often low, whereas the costs may be high. This paper is well-structured, and would provide excellent material for discussion in a Money and Banking class.

Böhme et al. clarify the early, current, and future uses of Bitcoin. Here students are able to consider not only how Bitcoin functions, but how it may impact financial institutions in the future. The authors’ comprehensive review of the risks involved in Bitcoin will also help students better understand the implications of the cryptocurrency and the blockchain. Luther, in highlighting the regulatory costs and benefits associated with Bitcoin, provides students a framework to better understand the inherent obstacles of Bitcoin. Both Böhme et al. and Luther (2016b) aid students in considering the inquiry-based goal of this section.

---

11 See Weiser (2015) and Wong (2014) for more information.
Part III: Impactful Changes for the Central Bank

A large portion of any Money and Banking course is dedicated to the role of the central bank. Students study the structure, regulatory power, and monetary policy of the central bank. Bitcoin’s lack of a central authority leaves it highly unregulated at present. It is also somewhat unclear how policy makers would conduct monetary policy in a world dominated by bitcoin. To conclude the study of cryptocurrencies, we suggest introducing these questions from a strictly inquiry-based learning approach. After all, there are not direct answers for the questions posed.

Armed with a solid understanding of how cryptocurrencies work, as well as the potential implications they may have on the banking system, students should approach this third section prepared to discuss. They will consider the future of the currency as well as the potential for technological innovation based on the blockchain protocol. This newfound competence enables a robust discussion regarding the future of regulation and monetary policy in a world of cryptocurrencies. No one knows the answers to these questions, but simply raising them and allowing for discussion provides students an opportunity to display their critical thinking skills and analytic reasoning.

We suggest the review of two sources in conjunction with this section. First, the Bank for International Settlement produced a report in 2015 reviewing digital currencies. Approachable, yet thorough, this report provides students with not only a broad overview of the supply and demand structure of cryptocurrencies, but also a detailed consideration of the potential impact on central banks of widespread adoption of such currencies. Though the use of cryptocurrencies and blockchain technology is still in its infancy, the potential to disrupt the financial system is tremendous. The central bank could be affected in various ways. This report reviews the impact on financial stability and monetary policy.

Second, we suggest using Raskin and Yermack’s 2016 NBER working paper, “Digital Currencies, Decentralized Ledgers and the Future of Central Banking” which promotes thoughtful consideration of our inquiry-based goals. The authors suggest that the introduction of algorithmic digital currencies and blockchain technology will force central banks to adapt. Raskin and Yermack detail three distinct possibilities for central banks in relation to cryptocurrencies and blockchain technology. The first consideration is that competition that could arise between an autonomous digital currency, such as Bitcoin, and the national currency. The second examines the implications of a digital currency issued by the central bank. Finally, the authors consider the operation of monetary policy using blockchain technology. A careful review and discussion of these potential changes will lead to robust classroom discussion and achievement of our inquiry-based goal.

Bitcoin was designed to be a self-sustaining currency. Yet, by definition, exercising monetary policy leads to the control of the money supply by policy makers. Is this possible in a world of cryptocurrencies? This is a question that needs to be explored in much more detail by researchers, policy makers, and students alike.

Student Survey and Results

We utilized the methods outlined above while teaching our Money and Banking courses during the spring of 2015, which were comprised of sixty-two students across three sections at two different liberal arts colleges. At the end of the semester, we each surveyed our students to see what they thought of the material that we incorporated from outside of the textbook, focusing on cryptocurrencies. In Table 3, you can find our results from the survey.

In developing the survey, we aimed to measure the effectiveness of the course and the value that it brought to our students. Overall, students’ responses indicate that they strongly valued and enjoyed discussion surrounding cryptocurrencies. They understood the importance of this discussion and believed that it should be part of today’s Money and Banking courses.

---

12 Some of the sources outlined were not utilized when the survey was implemented as they have been published since our courses were offered. We plan to add these articles the next time we teach Money and Banking.
Table 3: Student Survey Results

<table>
<thead>
<tr>
<th>Question</th>
<th>Mean: Scale of 1 to 5&lt;sup&gt;13&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>The discussion of virtual currencies and virtual banking&lt;sup&gt;14&lt;/sup&gt; was relevant to the course material.</td>
<td>4.4516 (0.6188)&lt;sup&gt;15&lt;/sup&gt;</td>
</tr>
<tr>
<td>My awareness of virtual currencies and virtual banking has increased as a result of this course.</td>
<td>4.4516 (0.7394)</td>
</tr>
<tr>
<td>My interest in virtual currencies and virtual banking has increased as a result of this course.</td>
<td>4.0806 (1.0130)</td>
</tr>
<tr>
<td>My understanding of virtual currencies and virtual banking has increased as a result of this course.</td>
<td>4.3548 (0.7487)</td>
</tr>
<tr>
<td>The instructor effectively integrated the virtual currencies and banking materials with the text material.</td>
<td>4.2903 (0.7103)</td>
</tr>
<tr>
<td>The assigned readings, specifically “Bitcoin as money?” by Lo and Wang at the Federal Reserve Bank of Boston and “The Economic Organisation of a P.O.W. Camp” by R.A. Radford, were relevant and aided in my understanding of the material.</td>
<td>4.4344 (0.6675)</td>
</tr>
</tbody>
</table>

N = 62

Conclusion

Seeking to fill a void where discussion of cryptocurrencies is either briefly introduced and then disregarded, or completely nonexistent in many modern Money and Banking courses, we sought to develop methods to expose our students to this important financial innovation. We have explained the motivation for why students need exposure to cryptocurrencies and blockchain technology in an academic setting. We have provided an overview of cryptocurrencies, paying special attention to Bitcoin. We then offered a plan for discussing this important topic with students alongside the material already presented in Money and Banking courses. In doing so, we survey recommended academic articles, suggest inquiry-based goals, and pose useful discussion questions for the classroom. Based on responses from our own students, the approach outlined herein is largely considered effective and valuable. We hope others will build on our approach to introduce students to the exciting—and promising—financial innovation of cryptocurrencies.

References


<sup>13</sup> The following scale was utilized: Strongly Agree (5), Agree (4), Neither Agree nor Disagree (3), Disagree (2), and Strongly Disagree (1).

<sup>14</sup> When we taught the course we included the discussion of virtual banking and mobile payment systems; however, we decided to focus our efforts of this paper towards the importance of cryptocurrencies.

<sup>15</sup> Standard deviations reported in parentheses.


The Benefits of Recognizing Interest Rates Independent of Goods Market Equilibrium

S. Kirk Elwood1

Abstract

The macroeconomic theory presented in undergraduate textbooks commonly characterizes goods market equilibrium and the equilibrium interest rate as jointly determined, which is conspicuously at odds with the high frequency at which market interest rates are observed and the lower frequency at which the goods market is thought to clear. One might conclude that the goods market is not necessarily in equilibrium during periods when equilibrium interest rates prevail, however the theory generally gives no attention to nonequilibrium situations. This paper argues that openly recognizing interest rates as independent of goods market equilibrium helps convey how aggregate demand shocks affect output.

Introduction

The macroeconomic theory presented in undergraduate textbooks relies heavily on the concept of goods market equilibrium. It is usually first presented as occurring in a closed economy when desired saving ($S_d$) equals desired investment ($I_d$) and is depicted in a diagram with the saving and investment schedules in which it is assumed equilibrium forces push the interest rate to deliver goods market equilibrium. Thus, goods market equilibrium and the equilibrium interest rate are presented as jointly determined.2

Goods market equilibrium is revisited with the Keynesian Cross model, but in a context where the interest rate isn’t specified and the level of desired (or “planned”) investment is exogenous.3 Most intermediate macroeconomic textbooks go on to present the IS curve which again strictly associates goods market equilibrium with an equilibrating interest rate, i.e., the IS curve shows the interest rate that equilibrates the goods market at each level of output.

Since the approach based on the saving and investment schedules as well as the IS curve both have goods market equilibrium and the equilibrium interest rate jointly determined, they both present the two equilibria as occurring at the same frequency. However, interest rates are observed at a high frequency (i.e., new interest rates are observed every few seconds) and economists do not dispute that the observations are equilibrium rates. The combination of these conditions would seem to assert that the goods market clears at the same high frequency as equilibrium interest rates occur, but this is rarely explicitly stated. If true, it would mean that the goods market is clearing almost continuously, without any undesired production (and, therefore, undesired changes in inventories) for more than a mere moment.

The alternative is to maintain that the goods market clears at a lower frequency so that its equilibrium status is – at the very least – uncertain during periods when equilibrium interest rates are

1 Professor of Economics, James Madison University, Harrisonburg VA 22807. email:elwoodsk@jmu.edu. The author would like to thank two anonymous referees for many valuable comments on an earlier draft.

2 The open economy version of goods market equilibrium (i.e., $S^d = I^d + CA$, where $CA$ is the current account) usually follows in a later chapter where, once again, goods market equilibrium and the equilibrium interest rate in a large open economy are determined together (the interest rate in a small open economy is exogenous).

3 The Keynesian Cross framework specifies goods market equilibrium as output ($Y$) equaling expenditures ($E = C^d + I^d + G$ for a closed economy where $C^d$ is desired consumption and $G$ is government purchases). The equivalency of $Y = E$ and $S^d = I^d$ is shown by starting with $S^d = Y - C^d - G$ and substituting $I^d$ for $S^d$, which can be written as $Y = C^d + I^d + G \equiv E$. It is common for presentations of the Keynesian Cross to assume that $C^d = C$. 
observed. But such acknowledgments of possible nonequilibrium situations are unusual in today’s mainstream macroeconomic theory, just as the word “disequilibrium” is rarely found in the recent mainstream macroeconomic literature. But if equilibrium interest rates observed at high frequency are considered worthy of notice, is it consistent to dismiss conditions in the goods market at the same frequency – whether it is in equilibrium or not – especially given that the interest rate and goods market are so theoretically intertwined with one another?

It is notable that – just as with the saving and investment schedule portrayal of goods market equilibrium – presentations of the IS curve do not seriously consider the possibility that the goods market is not in equilibrium. This is convenient because there are two alternative approaches to developing the IS curve that suggest very different equilibrating forces. The first approach constructs the IS curve using the investment schedule and the Keynesian Cross model. It perceives a change in the interest rate as altering desired investment and, therefore, (desired) expenditures. The deviation of expenditures from output then causes unwanted changes in inventories which brings about changes in output until it matches the level of expenditures and, therefore, goods market equilibrium is achieved (e.g., for this derivation of the IS curve see Blanchard, 2017, or Dornbusch et al., 2014). The alternative approach constructs the IS curve based directly on the saving and investment schedules: A change in output shifts the saving schedule (along the investment schedule) and causes goods market equilibrium to occur at a new equilibrium interest rate (e.g., this IS curve is found in Abel et al., 2017, or Mishkin, 2015).

So while the first rendering of the IS curve imagines changes in the interest rate precipitating changes in the goods market equilibrium output, the second considers changes in output that lead to new equilibrating interest rates. These different interpretations relate completely different responses to a lack of equilibrium: The first maintains that it is output that adjusts to bring about goods market equilibrium, while the second perceives interest rates adjusting to attain it. However, these different reactions are not mutually exclusive as will be shown in the discussion of the forces that are responsible for goods market equilibrium below.

Although the notion of disequilibrium has been conspicuously absent from mainstream macroeconomic discussions for nearly three decades, it has received serious attention. In the 1960’s Don Patinkin, Robert Clower, and Axel Leijonhufvud all successfully advanced the idea of macroeconomic disequilibrium. Barro and Grossman (1971) introduced a widely cited “general disequilibrium” model, but then interest in this branch of theory waned as mainstream macroeconomic theory turned to microfoundations and market clearing models (see Kydland and Prescott, 1982, and Long and Plosser, 1983, for two highly influential early examples). The workhorse models became the new dynamic stochastic general equilibrium (DSGE) models that either specify general equilibrium or, as with New Keynesian DSGE specifications, assumed market clearing in all markets except for one where a nominal rigidity is imposed. The new microfoundations and market clearing standards effectively pushed aside macroeconomic disequilibrium as a competing characterization of the complicated nature of macroeconomic activity in the mainstream literature.

Disequilibrium has (implicitly) remained part of the Austrian literature which conceives of markets as processes and is more concerned with how an equilibrium is approached than its possible realization. It characterizes markets as inherently dynamic processes driven by the actions of their participants, whose competing behavior over time leads to the discovery of optimal prices (e.g., see Hayek, 1968, or Horwitz, 2009). However, this perspective is outside the more orthodox macroeconomic view presented in undergraduate textbooks.

This paper does not attempt to engage in the argument that understanding the workings of macroeconomic disequilibrium is key to grasping an economy’s macroeconomic performance. Rather, it merely highlights the important forces that a lack of equilibrium engenders, which is a much less involved task. Admittedly, the use of the word “disequilibrium” threatens to conjure up connotations of the bigger

---

4 The word “disequilibrium” has not shown up in the abstract of an American Economic Review article since (at least) 1986, whereas “equilibrium” has been in the abstracts of 371 articles. Since 1994 “disequilibrium” was in one Journal of Economic Perspectives’ abstract, while “equilibrium” has been in 40. More directly relevant to this paper is that the word “disequilibrium” is rarely found in undergraduate macroeconomic textbooks, even those that discuss goods market equilibrium (e.g., Abel et al., 2017, Blanchard, 2017, Dornbusch et al., 2014, Mankiw, 2016, and Mishkin, 2015). Hubbard et al. (2014) briefly mention “disequilibrium” just to dismiss any notion that it significantly influences macroeconomic activity.

argument. This paper will rely on the expressions “nonequilibrium” or “lack of equilibrium” in the hope of confining the analysis to the forces that arise to bring about equilibrium. In fact, the approach could be viewed as consistent with the equilibrium focus since the recognized lack of equilibrium primarily serves to explain how the economy is driven towards equilibrium.

Precedence for the recognition of the lack of equilibria and its resulting forces in introductory economics courses is easily found in introductory microeconomics classes where nonequilibrium market prices are shown to produce either shortages (a.k.a. excess demands) or surpluses (a.k.a. excess supplies), and the forces that move prices to their equilibrium values are explained. What is being advocated in this paper with respect to interest rates and the goods market is completely analogous.

The difficulty in accepting that goods market equilibrium and the equilibrium interest rate are jointly determined is not allayed by the available data. Measures of output (and saving and investment) are involved collections of aggregated data at monthly or quarterly intervals, whereas market determined interest rates are easily observed by the moment. The precise and regularly reported interest rates seem completely incongruous with theory that presents interest rates as achieving equilibrium jointly with something as difficult to measure and perceive as goods market equilibrium. It asks a lot of those learning economics to approach interest rates with theory that can only explain rough interest rate trends at best and has no capacity for capturing interest rates at their observed frequency, especially when a more cogent and practical theory of interest rate determination is available (as will be shown below).

The problem with pinning down goods market equilibrium in the actual economy (despite clearly observed – supposedly jointly determined – equilibrium interest rates) juxtaposed with macroeconomists’ reluctance to explicitly recognize the possibility or significance of a lack of equilibrium in that same market suggests incomplete if not inconsistent thinking. This paper offers a simple, yet more coherent perspective on interest rates as well as on output by openly acknowledging the possible lack of equilibrium in the goods market for a given equilibrium interest rate.

Conceding a lack of equilibrium in the goods market generates benefits beyond freeing interest rate theory to more accurately capture the forces that bring about interest rates at their observed high frequency. A second advantage of the approach is a more thorough narrative of the effects of aggregate demand shocks – including both fiscal and monetary policy shocks – on the economy. Whereas the traditional tactic is to jump to the new equilibrium resulting from a shock, the approach presented here directly shows how a shock puts the economy out of equilibrium and then, importantly, explicitly notes the forces that propel the economy to its new equilibrium. While the distinction may seem trivial to those whose primary concern is identifying and comparing equilibria (i.e., comparative static analysis), the approach relates a process by which new equilibria are achieved as opposed to simply assuming their ultimate realization.

This paper proceeds by presenting a straightforward theory of interest rate determination that is uncontroversial, yet either gets little attention or is completely ignored by macroeconomic textbooks precisely because it specifies an equilibrium interest rate that is seemingly unrelated to the interest rate they present as jointly determined with equilibrium in the goods market. Next, that equilibrium interest rate will be shown to not just coexist with, but to be responsible for, a lack of equilibrium in the goods market. The resulting forces working to restore goods market equilibrium (in both the short run and long run settings) will be detailed, including how those forces alter the equilibrium interest rate. Finally, the macroeconomic effects of fiscal policy, monetary policy, and other types of macroeconomic shocks will be examined, where it is demonstrated that the approach’s recognition of a lack of equilibrium in the goods market enables a more graphic depiction of how shocks affect output.

**The Price of Bonds and the Interest Rate**

Although the interdependence of interest rates and output (as well as consumption, investment and international capital and trade flows) have led to theories in which interest rates are determined jointly with output, it remains indisputable that interest rates correspond to bond prices as determined by the large and active bond markets (as depicted by Figure 1). The market price for a bond’s promised stream of future receipts identifies an interest rate (or “yield” or “rate of return”) on that bond, and increases (decreases) in the bond’s price means lower (higher) interest rates.
It is true that many textbooks point out this very relationship between bond prices and interest rates and even give examples of how bond prices translate to interest rates. To accept that the prices of bonds reported each moment from the bond markets are equilibrium prices is to accept that equilibrium interest rates are observed at that frequency as well. But textbooks rarely take the next (seemingly logical) step of presenting the bond market model as the basis for a theory of equilibrium interest rates.6

Many textbooks do provide liquidity preference theory7 which strongly mirrors the bond market approach to interest rates. It specifies money demand rising and falling which is equivalent to the demand for nonmonetary assets (including bonds) falling and rising respectively (assuming constant wealth), so that interest rates and bond prices move in the appropriate opposite directions. But the idea that equilibrium interest rates occur at a high frequency is not normally mentioned. In textbooks where the LM curve is then introduced – which is universally developed from liquidity preference theory – the focus is on the intersection of the LM with the IS curve and, therefore, where goods market equilibrium and the equilibrium interest rate are jointly determined.8

Many textbooks also present the loanable funds market as a theory of interest rate determination where borrowers demand loanable funds, lenders supply them, and the market price is the interest rate that adjusts to bring these flows into equilibrium.9 This theory’s interest rate differs from the interest rate that clears the goods market since new bank credit contributes to the supply of loanable funds but not saving. In addition to bank credit being a reason why the loanable funds determined interest rate differs from the goods market equilibrating rate, both theories share a common weakness: They only consider flows of either newly issued credit or saving and disregard the influence of the stock of existing debt (i.e., bonds) at the beginning of the period over which the flows take place. Worded differently, the loanable funds determination of interest rates as well as the desired saving and investment determination only account for the primary bond market and do not acknowledge the massive secondary bond market, despite newly issued bonds and existing bonds being near (if not perfect) substitutes for each other. The bond market approach to interest rates (via bond prices) accommodates both the primary and secondary bond markets and, therefore, does not suffer this same problem.

The bond market model is powerful at displaying how events affect the interest rate. For example, a government deficit financed by bond sales will increase the supply of bonds. The shift in the supply curve to the right will cause the price of bonds to fall and interest rates to rise accordingly. As another example, expansionary monetary policy in the form of open market purchases or quantitative easing increases the demand for bonds (by the Fed), causing bond prices to rise and interest rates to fall. A further example: A

---

6 Introductory macroeconomic textbooks that clearly discuss the inverse relationship between bond prices and interest rates without entertaining the idea of bond markets determining interest rates include Colander (2013) and Frank and Bernanke (2016). Intermediate texts include Abel et al. (2017), Blanchard (2017), and Dornbusch et al. (2014). Hall and Lieberman (2013, see pages 400-401) are an exception in that they discuss how bond market activity influences short run interest rates.

7 For example, see Frank and Bernanke (2016), Hall and Lieberman (2013), and Mankiw (2018), although only the latter explicitly refers to it as liquidity preference theory (i.e., the others show the same money demand/supply determination of the interest rate without using the theory’s traditional name).

8 Textbooks that build upon liquidity preference theory to construct the LM curve of the IS/LM model include Abel et al. (2017), Hubbard et al. (2014), and Mankiw (2016).

9 Typical introductory textbook presentations of loanable funds theory can be found in Hall and Lieberman (2013), Hubbard and O’Brien (2015), and Mankiw (2018).
decrease in the expected future marginal product of capital would shift the supply of bonds to the left as there would be less borrowing and fewer bonds issued. This would increase bond prices and lower interest rates. Yet another example is an increase in the demand for bonds due to increased riskiness of equities, which causes bond prices to rise and, therefore, interest rates to fall.

This simple theory regarding the determination of equilibrium bond prices and, therefore, market determined equilibrium interest rates is always applicable. This is not to say that other ways of thinking about interest rates are not legitimate, but any further meaningful theoretical perspectives on market determined interest rates should be consistent with this bond market foundation. It is worth repeating that while textbooks note the relationship between bond prices and interest rates, few present the bond market model as a theory of interest rate determination (see Footnote 6 above).

**Goods Market Equilibrium**

Goods market equilibrium exists in a closed economy when $S_d = I_d$ or, equivalently (as shown in Footnote 3 above), the amount produced ($Y$) is equal to the amount of desired expenditures within the economy ($E$). Since $S_d$ is a positive function of the real interest rate ($r$) and desired investment is a negative function,$^{10}$ one can present the saving and desired investment schedules and the resulting equilibrium $r (r^*)$ that provides goods market equilibrium (see Figure 2). But if one accepts that equilibrium real interest rates (which are equilibrium nominal interest rates corrected for expected inflation) are determined by the bond markets at a higher frequency than the goods market clears, observed interest rates at many moments would not necessarily be expected to be consistent with goods market equilibrium. How should this lack of equilibrium in the goods market be handled?

The microeconomic approach to introducing and justifying the concept of an equilibrium price offers guidance in this case. Specifically, those teaching microeconomics acknowledge the process by which prices are pushed from nonequilibrium to equilibrium values: A price below (above) the equilibrium price brings about a shortage (surplus) that causes sellers to raise (lower) the price. Note that the concept of not being in equilibrium is implicitly embraced in this account since the forces that are deemed responsible for pushing prices towards their equilibrium values only exist out of equilibrium. Why doesn’t macroeconomics regard the interest rate in the context of goods market equilibrium in a comparable fashion and, therefore, identify the forces at work when the equilibrium interest rate is not consistent with goods market equilibrium? This paper maintains that articulating these forces is helpful to understanding goods market equilibrium in the same way that knowing how equilibrium prices are derived improves the understanding of markets.

It is important to affirm that the distinction being examined between the bond market’s interest rate and the interest rate that equilibrates the goods market is the difference in the rate at which the two markets clear.$^{11}$ Just as a shock can cause the price of a widget to be higher than its equilibrium price before the existing surplus has had its full effect on the widget market, an equilibrium interest rate at a given moment can deviate from its goods market equilibrating value before the goods market has had sufficient time to clear. Once it is accepted that an observed interest rate might not be consistent with goods market

---

10 Since a higher interest rate means a higher opportunity cost of investing, the desired investment schedule is downward sloping.

11 Other distinctions clearly exist such as the bond market approach’s implicit accommodation of the influence of the secondary bond market. Similarities also exist including their representation of borrowing and lending activity.
equilibrium, then it naturally follows to consider the resulting forces that deliver goods market equilibrium. Even though designating an equilibrium may indirectly acknowledge the possibility of not being in equilibrium as well as the forces that arise when not in equilibrium, it is clarifying to explicitly identify those forces.

The two different constructions of the IS curve discussed above suggest different equilibrating forces promoting goods market equilibrium. However, there is no reason why these forces would be mutually exclusive and not both pushing the economy towards equilibrium. For example, consider the situation in which the interest rate is too high for goods market equilibrium (see Figure 3). The first IS curve story has the excess of $S^d$ over $I^d$—i.e., the excess assets demanded by savers over the assets intended to be created at the given interest rate—bid up the price of assets. This would drive the economy towards goods market equilibrium in two ways: The higher prices of bonds would mean lower interest rates that would move the economy towards goods market equilibrium, and, the higher asset prices would increase wealth in the economy (in the form of capital gains). The increase in wealth would increase consumption and reduce saving as per the permanent income hypothesis (shifting the $S^d$ curve to the left in Figure 3) again moving the economy towards goods market equilibrium. The fact that both $S^d$ and $I^d$ are flows means that the excess $S^d$ will promote increasing asset prices as a process that occurs over time as opposed to their prices rising instantaneously.

The second IS curve story maintains that $S^d > I^d$ means that $Y > E$ and, therefore, inventories are building up in an undesired way. The unintended increases in inventories promote cutting production so that $Y$ falls. This decrease in $Y$ then shifts the $S$ schedule to the left, i.e., towards goods market equilibrium. Again, this particular equilibrating force depends on changes in flows of $Y$ and $S^d$ which are not immediate.

Although the responses of an economy to an interest rate that is too low for goods market equilibrium mirror those of one with an interest rate that is too high, they are sufficiently important to state as well. The low interest rate means that $I^d > S^d$ and, therefore, the effort to invest more than what is saved leads to people attempting to finance the extra investment by selling assets. This could occur because the expected gains from the investment is thought to be worth selling existing assets to finance it, or, perhaps more commonly, many of the assets sold are newly issued bonds as people borrow to finance the investments. The process of selling both existing and newly issued bonds will drive down their prices and raise interest rates which, under these circumstances, will move the economy towards goods market equilibrium. Also, the same fall in asset prices constitutes a negative wealth shock that would reduce

---

12 It is important to note in this situation that people’s purchase of assets beyond what is needed to finance the new desired investment does not constitute investment (i.e., the creation of new capital) nor saving, but merely a swapping of assets (money for the nonmonetary asset) amongst members of the macroeconomy. Therefore, it might seem macroeconomically irrelevant. But the effect on asset prices and, therefore, interest rates should not be ignored.

13 Activity in services where inventories are not possible can also exert similar forces on output. For example, if expenditures on a service fall then those hired to provide the service can experience idle moments. Those idle moments can cause employers to release the superfluous employees and cause GDP to fall. Symmetrically, if expenditures on services increase it can induce the hiring of more service providers and greater measured output.
consumption and increase saving and, therefore, shift the saving schedule out towards goods market equilibrium.14

As with an interest rate too high for goods market equilibrium, there is the third equilibrating force. If the interest rate is so low that \( I^d > S^d \) and, equivalently, \( E>Y \), then inventories are depleting in an undesirable way. This induces greater production to replenish inventories and \( Y \) rises. This, in turn, causes the saving schedule to shift to the right towards goods market equilibrium.

To summarize, failure to be in goods market equilibrium generates multiple forces that work to bring about goods market equilibrium by influencing \( r \) as well as the levels of wealth and \( Y \) that both shift the saving schedule, where the respective contributions of the three to the attainment of goods market equilibrium likely varies with circumstances. Of course, it is the effect that the lack of goods market equilibrium has on \( Y \) that makes understanding the forces that equilibrate the goods market particularly valuable.

**Long Run Price Adjustment and Goods Market Equilibrium**

It is important to notice that the analysis so far has not factored in the economy’s full-employment level of output (\( \tilde{Y} \)) or the possibility of changes in the price level (P). Macroeconomic theory considers the output that exists under goods market equilibrium (i.e., \( Y \)) to be a “short run” equilibrium precisely because it is somewhat independent of what \( \tilde{Y} \) may be. More formally, the “short run” is defined as the period over which prices are too sticky to adjust and lead \( Y \) towards \( \tilde{Y} \), thus allowing \( Y \) to remain distinct from \( \tilde{Y} \). Short run output is commonly characterized by textbooks that present the AD/AS model by the intersection of the AD curve and the short run aggregate supply curve. Theory then designates the “long run” as the length of time required for the economy’s prices to adjust – i.e. for the short run aggregate supply curve to shift – so that \( Y \) attains \( \tilde{Y} \).15 For example, consider the case in which goods market equilibrium occurs at a level of \( Y \) below the economy’s \( \tilde{Y} \), i.e., the economy is in what many would refer to as a recession. The standard reasoning maintains that recessions lead to falling wages and prices as unhappily unemployed workers submit to lower wage offers and sellers lower prices to move unsold product, which shifts the short run aggregate supply down along the AD curve to higher levels of \( Y \) until \( \tilde{Y} \) is reached.

Even though goods market equilibrium is classified as a short run phenomena, the process by which an economy approaches long run equilibrium can still be viewed as a process in which price adjustments push the economy away from goods market equilibrium, which then unleashes those forces that act to reinstate goods market equilibrium until goods market equilibrium is established at the LR equilibrium level of output.

More specifically, the drop in P (or, more commonly, P not rising as much as expected) disrupts goods market equilibrium via two channels. First, a fall in P produces a “liquidity effect” (or “Keynes effect”), i.e., it increases the level of real balances (which equals the nominal money supply divided by P). The larger volume of real balances circulating around the economy is commonly believed to boost the demand for everything including bonds. The resulting higher bond prices and corresponding lower interest rates will cause \( I^d > S^d \) and unleash the three equilibrating forces described above. The one resulting from the undesired drop in inventories (since \( E>Y \)) causes \( Y \) to increase towards \( \tilde{Y} \). It is also true that the rising bond prices increases wealth, which would shift the saving schedule to the left and exacerbate the drop in desired inventories and, therefore, have an even greater adjustment of \( Y \).

The second way that the fall in P moves a country out of goods market equilibrium is through a further wealth effect known as the “Pigou effect”: The drop in P increases the purchasing power of nominal balances (and expected receipts from government securities) that increase people’s real wealth, which will

---

14 While the possibility of new bonds being issued might suggest an increase in wealth that could overpower the decreased value of existing assets enough to cause overall wealth to rise, there are two reasons why this is not so. First, the sale of newly issued bonds could not alter wealth since they are sold for an equal value of money. Additionally, private bonds are not generally thought to affect aggregate wealth since the liability they represent to the seller equals their asset value to the buyer. Importantly, it is under the assumption that bonds and equities serve as substitute assets that causes lower bond prices to lower equity prices and reduce wealth.

15 Introductory textbooks that present the AD/AS model this way include Colander (2013, see pages 195-198), Hall and Lieberman (2013, see pages 444-449) and Mankiw (2018, see pages 720-724). Intermediate macroeconomic textbooks that present the IS/LM model then use it to derive the adjusting prices that shift the LM curve so as to bring about a goods market equilibrium that coincides with \( \tilde{Y} \) (e.g., see Abel et al., 2017, pages 340-341, and Mankiw, 2016, pages 349-350). Although this price adjustment story is commonly related by textbooks to explain why economies gravitate to full employment, problems with the approach have been noted over the years, e.g., see Colander, 1995)
also increase consumption and shift back the saving schedule. Whereas it might seem unintuitive for there to be positive wealth effects during a recession, note that they are not coincident with the shock that causes the recession, but part of the process of recovering from recession.

While this explains how an economy in recession naturally approaches $\bar{Y}$, a comparable explanation of how a booming economy settles back to $\bar{Y}$ simply requires symmetric reasoning. So, more generally, P adjustments in the long run will move the economy to a particular goods market equilibrium where $Y$ equals $\bar{Y}$.

Because goods market equilibrium is theoretically conceived as a condition that determines short run equilibrium $Y$ and is independent of the determination of $\bar{Y}$, shocks that affect $\bar{Y}$ – which are commonly referred to as aggregate supply shocks – do not directly disturb goods market equilibrium. Instead, the standard textbook approach presents aggregate supply shocks as altering $\bar{Y}$ and, therefore, shifting the AS curve and causing a change in P which (as described above) leads the economy to a new goods market equilibrium. For example, a negative supply shock shifts the AS curve to the left causing higher P that precipitates the drop in $Y$ to a new goods market equilibrium, i.e., the AS curve shifts along the AD curve.\(^{16}\) In contrast, there are shocks to AD that directly disrupt goods market equilibrium and lead to new levels of short run equilibrium output without altering $\bar{Y}$. It is these aggregate demand shocks that will be examined in the next section of this paper.

The discussion in the preceding section showed that the absence of goods market equilibrium not only means that $Y$ is moving in the direction of a short run equilibrium, but that the interest rate is adjusting with it: The lack of goods market equilibrium precipitates activity in the asset markets that alters bond prices and, therefore, the interest rate that helps reinstate goods market equilibrium. Thus, while an “equilibrium” interest rate may prevail at a high frequency as determined by the bond market, that interest rate continues to be influenced by a lack of equilibrium in the goods market until goods market equilibrium also occurs, which only happens at a much lower frequency. The value of highlighting the equilibrating forces that act upon output as well as the interest rate when the goods market is not in equilibrium is a major focus of this paper. Now this section has shown that the interest rate is subject to still other equilibrating pressures until it is consistent with goods market equilibrium at full employment, which occurs at an even lower frequency. Clearly understanding and communicating the different realizations and phases of the interest rate is not an insignificant challenge.

Macroeconomics already struggles with the concepts of “short run equilibrium output” and “long run equilibrium output” which, though accepted by most economists in theory, are often controversial when used to describe observed conditions. The same issues are present with the ideas of an interest rate that provides goods market equilibrium consistent with short run equilibrium output and an interest rate under (goods market equilibrium and) long run equilibrium output. Then, in addition, there is the equilibrium interest rate observed at a high frequency that is determined by the bond market. It all creates a challenge to accurately characterize the interest rate under the different conditions, such as appropriately labeling this latter equilibrium that occurs in less than the short run.

### Analysis of Shocks that Disrupt Goods Market Equilibrium and Affect Output

Now that the relationship between goods market equilibrium and interest rates has been presented, the effect of shocks that disturb goods market equilibrium can be analyzed in more detail. Again, the set of shocks to consider belong to the family of aggregate demand shocks – including fiscal and monetary policy shocks – since they are essentially defined by their impact on goods market equilibrium output assuming P has not yet fully adjusted. The analysis is not applicable to aggregate supply shocks since they have no direct impact on the current goods market equilibrium, but only have long run effects from the changes in P that result from the shock to $\bar{Y}$ (as was discussed above).

The analysis provided below will focus on how each type of shock affects the saving and/or investment schedules and/or the (real) interest rate so as to determine whether the interest rate has become too high or too low to provide goods market equilibrium. From that point the return to goods market equilibrium is carried out by the equilibrating forces presented in the preceding section.

Consider the effects on an economy that is initially in goods market equilibrium of:

---

\(^{16}\) Examples of this characterization of aggregate supply shocks can be found in Abel et al. (2017, see pages 309-310), Hall and Lieberman (2013, see pages 449-450), and Mankiw (2018, see pages 712-722).
1) expansionary monetary policy: The expansion of the monetary base by central bank purchases of bonds will lower interest rates as it raises bond prices. Not only will the lower interest rates cause $I^d$ to exceed $S^d$, but the shortage of saving will be exacerbated by the positive wealth effect from the higher bond prices that will shift the saving schedule in (see Figure 4). The resulting equilibrium forces detailed above would then push the economy back to a new goods market equilibrium. Specifically, the excess $I^d$ would lead to sales of newly issued and existing bonds to finance the desired investment and drive bond prices down and, therefore, interest rates up towards goods market equilibrium. Additionally, the lower bond (and other asset) prices would constitute capital losses that would lower wealth and promote more saving (shifting out the $S^d$ schedule and, therefore, helping restore goods market equilibrium). Finally, any amount of $I^d$ that exceeds $S^d$ would see undesired decreases in inventories that would lead to increased production and output. The resulting increase in output would also promote equilibrium by shifting out the saving schedule.

![Figure 4](image)

2) increase in investor confidence (i.e., an increase in the expected future marginal productivity of capital, or, an increase in “animal spirits”): An increase in investor confidence will shift the $I^d$ curve out, causing $I^d$ to exceed $S^d$ at the going interest rate. This would precipitate the issue and sale of new bonds as well as the sale of existing assets in the effort to finance the desired investment above that financed by desired saving, which would raise interest rates. The reduced asset prices would also constitute lower wealth (despite any issuing of new bonds as explained in Footnote 14) that would provide greater incentives to save so that the saving schedule shifts out. Thirdly, unplanned drops in inventories promote greater Y and the saving that accompanies it (shifting out the saving schedule further). All three of these effects would push the economy to a new goods market equilibrium.

3) an increase in consumer confidence (that increases the mpc): An increase in consumer confidence will reduce precautionary saving (shifting the saving schedule to the left) and lead to $I^d > S^d$ at the going interest rate. The selling of assets to finance the desired investment will lower bond prices and raise interest rates as well as lower asset prices more generally which reduces wealth and promotes saving. The third equilibrating force is the undesired fall in inventories (since E > Y) giving reason for greater production, and the greater Y will bring about even greater saving that helps return the economy to goods market equilibrium.

4) an increase in the demand for money, which occurs with several notable phenomena including decreased inflation expectations, an increase in the riskiness of nonmonetary assets, a decrease in the liquidity of nonmonetary assets, and an increase in the interest paid on demand deposits: An increase in money demand will cause people to sell nonmonetary assets to acquire the desired money balances and the resulting drop in bond prices and higher interest rates will cause $S^d$ to exceed $I^d$. The drop in wealth from lower nonmonetary asset prices will also promote more saving and exacerbate the surplus.

But then the purchasing of assets by the excess desired saving will drive up bond prices (making interest rates fall back) as well as the prices of other nonmonetary assets (causing capital gains that increase wealth and reduce saving, thereby shifting the saving schedule back in), both of which work to return goods market equilibrium. The third force influences output as the increase in unwanted
inventories causes firms to cut back production. The drop in Y, in turn, reduces saving (shifting the saving schedule to the left) and helps push the economy back to goods market equilibrium.

It should be noted that a central bank that targets interest rates will neutralize the effects of an increase in money demand by accommodating it with open market purchases that keep bond prices and interest rates from changing in the first place.

5) an increase in G: An increase in G will decrease saving (since \( S = Y - C - G \)) and shift the saving schedule in. If the increase in G is financed by new taxes then C would be reduced and dampen the decrease in S, but only under some extreme assumption (e.g., the mpc =1) would the taxes’ effect on C completely offset the increased G and keep S from falling. Accordingly, a tax financed increase in G would be expected to cause \( I^d > S^d \) at the going interest rate. From that baseline, increasing the share of the new G financed by bond sales instead of taxes would cause C to fall less so that S decreases even more (given the increase in G). It is true that shifting the financing from taxes to newly issued bonds would reduce bond prices and, therefore, raise interest rates. But the effect that the flow of new bonds to finance the G relative to the stock of nonmonetary assets that already exists would be small, so that the rise in the interest rate would be too small to overcome the effect of the decreased flow of saving and, therefore, not prevent \( I^d > S^d \) (i.e., the two effects would be as depicted by Figure 5). Of course, if the central bank targets interest rates then it would counter any pressure for the interest rate to increase. The lower asset prices would also cause a negative wealth effect that would promote saving, but it would only dampen the decrease in saving from the increase in G.

![Figure 5](image)

The result of \( I^d > S^d \) would bring into play the three goods market equilibrating forces: The selling of assets to finance the desired investment would raise interest rates, the lower asset prices that reduce wealth and promote more saving, and the undesired fall in inventories that gives rise to an increase in Y that spurs additional saving.

**Conclusion**

The macroeconomic theory presented in undergraduate textbooks relies on models in which output and interest rates are jointly determined and, therefore, specify their equilibria at the same frequency. Given that interest rates are observed at a high frequency along with the difficulty of arguing that those interest rates do not reflect equilibrium, fidelity to the theory asserts that output achieves equilibrium at the same high frequency. But few economists directly defend that position. The alternative is to maintain that goods market equilibrium does not exist over periods in which equilibrium interest rates are observed, which is problematic because the macroeconomic theory focuses on equilibria without acknowledging the condition of not being in equilibrium. This paper argues that breaking with this equilibrium mindset and openly recognizing the lack of equilibrium in the goods market – only as needed to motivate the kinds of macroeconomic forces that promote equilibrium – will improve the teaching and understanding of macroeconomic principles.

There are two reasons for those introducing macroeconomics to be explicit about equilibrium not prevailing in the goods market. The first is simply to keep the theory from seeming ridiculously detached
from reality. The interest rate is clearly observed changing within periods of just minutes or seconds while observations on output are only possible via data collected over one or three month periods. Therefore, a theoretical approach in which interest rates and output are jointly determined so that their equilibrium values are conceptualized at identical frequencies must seem – at the very least – odd to anybody being introduced to the theory.

The second much stronger reason for acknowledging the absence of equilibrium in the goods market is because it is precisely the lack of equilibrium that engenders the important forces that drive output as well as interest rates towards their new equilibrium levels. Accordingly, recognizing these forces helps one understand and appreciate exactly how shocks that disturb goods market equilibrium ultimately impact both output and interest rates. This paper has presented a straightforward way to openly characterize as well as illustrate the case where equilibrium does not exist in the goods market and to explain the resulting forces that move output and interest rates. The simple and intuitive approach relates a consistent and reasonable narrative of the process by which different types of shocks to aggregate demand affect the economy.

References


Credit Default Swap –Pricing Theory, Real Data Analysis and Classroom Applications Using Bloomberg Terminal

Yuan Wen1 * and Jacob Kinsella2

Abstract
The valuation of Credit default swaps (CDS) is intrinsically difficult given the confounding effects of the default probability, loss amount, recovery rate and timing of default. CDS pricing models contain high-level mathematics and statistics that are challenging for most undergraduate and MBA students. We introduce the basic CDS functions in the Bloomberg Terminal, aiming to help the students visualize the complicated concept of CDS. Furthermore, we use real data extracted from the Bloomberg terminal to illustrate the CDS pricing model of Hull and White (2000). Our paper can be used in an upper-division undergraduate Finance class or an MBA class.

Introduction
A credit default swap (CDS) is a derivatives instrument that provides insurance against the risk of a default by a debt security issuer. A CDS contract generally includes three parties: first the issuer of the debt security, second the buyer of the debt security, and then the third party, which is usually an insurance company or a large bank. The third party will sell a CDS to the buyer of the debt security. The CDS offers insurance to the buyer of the debt security in case the issuer is no longer able to pay. In the case of a default, the seller of the CDS is obligated to compensate the buyer for her loss.

An example of a CDS will help illustrate how the cash flows work. In this example, Company X is issuing a 10-year, 8% bond with a $10 million par value. Company Y has excess liquid funds, which are earning no interest at this time, and so they decide to buy Company X’s bond. Company X is given a rating of BB by a credit rating agency, and so Company Y thinks that it would be beneficial to seek a credit default swap from New National Bank. The contract is written up and states that for the entire duration of the bond’s life, Company Y will pay 1% of the face value to the bank. In return, the bank will offer insurance against Company X defaulting on their bond payment.

The notional value of a CDS refers to the face value of the underlying security. When looking at the premium that is paid by the buyer of the CDS to the seller, this amount is expressed as a proportion of the notional value of the contract in basis points. Gross notional value refers to the total amount of outstanding credit default swaps.

CDS can be written on loans or bonds. For simplicity, we only examine CDS written on bonds. If the reference entity (bond issuer) defaults at time \( t \) \((t \leq T, \text{ where } T \text{ is the maturity date})\), the CDS buyer will get a payment from the seller. This payment is referred to as the payoff from the CDS. The payoff from a CDS is usually different from the amount of the debt because the recovery rate is non-zero in most cases. When a bond defaults, bondholders will typically get part of their investment back from the liquidation of the issuer’s assets. According to Moody’s ultimate recovery database, the mean and median recovery rates for bonds are 37 percent and 24 percent, respectively3. The payoff from a CDS in the event of a default is

1 Assistant Professor of Finance, State University of New York at New Paltz, New Paltz, NY 12561. Email: weny@newpaltz.edu
* Corresponding author

2 MBA Candidate, State University of New York at New Paltz, New Paltz, NY 12561.

Designing Effective Examples of Optimal Capital Structure Estimation for the Intermediate Corporate Finance Course

Robert B. Burney

Abstract

Capital structure policy is a mandatory topic in courses in corporate finance. The level of detail presented depends on the level of the course in question. While the topic is typically presented in overview in introductory courses, intermediate and advanced courses frequently include estimation of an optimal capital structure. Numerical examples of optimal capital structure in popular textbooks are frequently based on discounted cash flow valuation and weighted average cost of capital estimates which vary based on the degree of financial leverage. This paper points out some potential pitfalls of such examples and provides suggestions for maximizing their benefit to students.

A Quick Review of Optimal Capital Structure Theory

Optimal capital structure theory is essentially an application of discounted cash flow analysis. In this framework, the value of any asset can be estimated as the sum of the present values of the expected future cash flows associated with the asset. Because there is an inverse relationship between present values and discount rates, the possibility exists for increasing value by reducing discount rates.

For an entire firm, the relevant cash flows are the firm’s Free Cash Flows (FCFs). An appropriate discount rate for the firm’s free cash flows is the firm’s Weighted Average Cost of Capital (WACC) which considers in weighted proportion the required rates of return of all investors who provide capital to the firm.

The funding mix which a firm chooses determines its WACC. As the funding mix changes, the WACC changes, leading to the possibility of finding a minimum WACC – and, therefore a maximum firm value. Because the risk of financial distress increases with the level of financial leverage, the required rates of return on the individual component costs of capital increase as financial leverage increases. Thus, although inclusion of relatively cheaper debt in the funding mix at first reduces the firm’s WACC, eventually the higher required rates of return overwhelm the advantage of cheaper debt funding as the firm reaches higher levels of financial leverage. This explanation is commonly referred to as the “tradeoff” theory of capital structure.

Although other factors such as agency relationships and degree of information asymmetry also play a role, the tradeoff relationship is generally accepted as the fundamental reason that capital structure policy is an important managerial consideration. The examples discussed in this paper are the types often used to demonstrate the tradeoff relationship.

Numerical Examples

While mathematical functions and their graphical representations are often used to explain the effect of financial leverage on firm value, students can gain additional insight from working through a detailed
mathematical estimation. Typically, percentage costs of debt are simply given in these types of examples. These costs of debt may or may not be explicitly linked to realistic debt yields as might be associated with a level of default risk as reflected in debt ratings. In such examples, the costs of equity at different levels of financial leverage are most often estimated using the adjustment of the firm’s stock’s beta using the Hamada model. This general approach is used by practitioners in actual valuations, so structuring classroom examples in this way provides students with a skill directly transferrable to the workplace.

Overall, these types of examples of the estimation of an optimal capital structure can be quite beneficial to the students. However, the precise structure used in the example must be carefully developed to not only demonstrate the key relationships, but also to increase the likelihood that a student will be able to work his or her way through the calculations in the example. Certain examples from popular textbooks inadvertently introduce potential pitfalls to the student which seem to consistently result in student frustration.

While many textbooks use some type of mathematical example when demonstrating the estimation of optimal capital structure, few provide as comprehensive an example as that presented in the Brigham group of intermediate and MBA level textbooks. Readers who teach corporate finance and have used one of the Brigham products are no doubt familiar with the now long-lived Strasburg Electronics Company example which has appeared largely unchanged in multiple versions and editions of the Brigham textbooks.

In this example, the hypothetical company is considering potential changes to its capital structure. An investment banking team is said to have provided the necessary inputs. The example then describes the process of estimating the firm’s WACC and value at various possible levels of financial leverage. For ease of calculation, the firm is assumed to have zero growth, so the necessary discounting can be accomplished using the simple present value of a perpetuity formula. In addition, the zero growth rate assumption makes the calculation of free cash flow particularly straightforward. In a zero growth situation, there is no required additional investment in operating working capital. And, since depreciation charges would approximate actual required investment in fixed assets, free cash flow would be equal to Net Operating Profit After Taxes (NOPAT). NOPAT is calculated as EBIT(1-T).

In the Strasburg example, Net Income is $50 million and free cash flow is $30 million. The required rates on debt are said to have been provided by the firm’s investment bankers. The rates on debt increase as financial leverage increases. The after-tax effective cost of debt is simply the pre-tax cost times the adjustment factor (1-T), where T is the marginal corporate tax rate. The firm’s required rate of return on equity capital is calculated using the Capital Asset Pricing Model (CAPM). Beta is adjusted for each potential level of financial leverage using the Hamada Model. Finally, the number of shares outstanding is calculated using the relationship between the value of the firm’s operations at each level of financial leverage and the presumed amount of additional debt funding necessary to create that level of leverage. The Brigham group’s example is presented in Table 1 below. In the table, “Shares Repurchased” is presented to aid in the subsequent discussion although this variable does not appear in the original Brigham example. Note that all dollar values are stated in terms of millions except for EPS and share value.

The required calculations are described in detail in the textbook and in a concise summary in the notes to the primary table showing the main results. For the most part, the calculations are relatively easy to follow. The one exception is the calculation necessary to determine the number of shares outstanding after the repurchase necessary to move the firm to the new level of financial leverage. The formula provided is as follows.

\[ n_{Post} = n_{Prior} \times \frac{(V_{OpNew} - D_{New})}{(V_{OpNew} - D_{Old})} \]  

(1)
In Table 1, the zero leverage value of beta is presented in bold. Also, the number of shares at the .20 total debt ratio level of financial leverage is presented in bold. If a student begins with the zero level of leverage, the numbers in the table will appear to differ from the number of shares resulting if one begins at the 12.72 million shares indicated at the zero financial leverage level. This is because the nature of the relationship causes the number of shares outstanding calculated from the anchored starting point of 10 million to differ from the number of shares resulting if one begins at the 12.72 million shares indicated at the zero level of leverage.

In the author’s experience, the motivated student who does attempt to prove the values in the table will begin at the zero level of financial leverage. He or she will be able to calculate the progressively larger betas that result as financial leverage increases. But, the student will not be able to replicate the number of shares outstanding. Nearly every semester, a student has approached the author to disclose the “error” in the table. Of course, the table is technically correct, but if a large percentage of the motivated students are unable to replicate the data in the table using the formulas given, there must be something about the table’s nature of the relationship causes the number of shares outstanding calculated from the anchored starting point of 10 million to differ from the number of shares resulting if one begins at the 12.72 million shares indicated at the zero financial leverage level.

\[ \beta_L = \beta_U [1 + (1 - T)(w_d/w_s)] \]

In this model, \( \beta_L \) is the firm’s beta when using financial leverage, while \( \beta_U \) is the firm’s beta when entirely equity financed. \( T \) is the corporate tax rate. \( w_d \) and \( w_s \) are the portions of funding coming from debt and equity, respectively.

Table 1: Brigham Textbook Group Example

<table>
<thead>
<tr>
<th>W_d</th>
<th>0</th>
<th>0.1</th>
<th>0.2*</th>
<th>0.3</th>
<th>0.4**</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>W_s</td>
<td>1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
</tbody>
</table>

| Pre-tax r_d (%) | 7.70 | 7.80 | 8.00 | 8.50 | 9.90 | 12.00 | 16.00 |
| After-tax r_d (%) | 4.62 | 4.68 | 4.80 | 5.10 | 5.94 | 7.20 | 9.60 |
| \( \beta \) | \textbf{1.0870} | 1.1594 | 1.2500 | 1.3665 | 1.5217 | 1.7391 | 2.0652 |
| \( r_m \) (%) | 12.30 | 12.30 | 12.30 | 12.30 | 12.30 | 12.30 | 12.30 |
| \( r_f \) (%) | 6.30 | 6.30 | 6.30 | 6.30 | 6.30 | 6.30 | 6.30 |
| \( r_s \) (%) | 12.82 | 13.26 | 13.80 | 14.50 | 15.43 | 16.73 | 18.69 |
| WACC (%) | 12.82 | 12.40 | 12.00 | 11.68 | 11.63 | 11.97 | 13.24 |
| Value of Operations | 233.98 | 241.96 | 250.00 | 256.87 | 257.86 | 250.68 | 226.65 |
| Value of Debt | 0.00 | 24.20 | 50.00 | 77.06 | 103.14 | 125.34 | 135.99 |
| Value of Equity | 233.98 | 217.76 | 200.00 | 179.81 | 154.72 | 125.34 | 90.66 |
| # Shares | 12.72 | 11.34 | \textbf{10.00} | 8.69 | 7.44 | 6.25 | 5.13 |
| Shares Repurchased*** | 1.37 | 1.34 | 1.31 | 1.25 | 1.20 | 1.11 |
| $ Value per Share | 18.40 | 19.20 | 20.00 | 20.69 | 20.79 | 20.07 | 17.66 |
| Net Income | 30.00 | 28.87 | 27.60 | 26.07 | 23.87 | 20.98 | 16.95 |
| $ EPS | 2.36 | 2.54 | 2.76 | 3.00 | 3.21 | 3.36 | 3.30 |

Notes: *starting capital structure, **optimal capital structure, ***not presented in original example.
layout which does not fit the typical student’s approach. Clearly a student with “Sequential Learner” tendencies will start with the zero level of leverage and attempt to work up.

Table 2 presents the variation in the example’s values which would result if one begins at the zero financial leverage and works his or her way up through the table as the level of financial leverage increases. In each case, the formulas are applied to a given case and the subsequent case in stepwise fashion. Thus, each estimate is based on the immediately preceding estimate – not on the zero values. The calculated values will be the same up to the number of shares. Thereafter, all resulting share related values will be different from the base case of the Brigham example. In Table 2 the values which are different are presented in italics.

Table 2: Variation One – Stepwise From Zero (Changes in Italics)

<table>
<thead>
<tr>
<th></th>
<th>0*</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4**</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_d$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$W_s$</td>
<td>1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
</tr>
<tr>
<td>Pre-tax $r_d$ (%)</td>
<td>7.70</td>
<td>7.80</td>
<td>8.00</td>
<td>8.50</td>
<td>9.90</td>
<td>12.00</td>
<td>16.00</td>
</tr>
<tr>
<td>After-tax $r_d$ (%)</td>
<td>4.62</td>
<td>4.68</td>
<td>4.80</td>
<td>5.10</td>
<td>5.94</td>
<td>7.20</td>
<td>9.60</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.0870</td>
<td>1.1594</td>
<td>1.2500</td>
<td>1.3665</td>
<td>1.5217</td>
<td>1.7391</td>
<td>2.0652</td>
</tr>
<tr>
<td>$r_m$ (%)</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
</tr>
<tr>
<td>$r_f$ (%)</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
</tr>
<tr>
<td>$r_s$ (%)</td>
<td>12.82</td>
<td>13.26</td>
<td>13.80</td>
<td>14.50</td>
<td>15.43</td>
<td>16.73</td>
<td>18.69</td>
</tr>
<tr>
<td>WACC (%)</td>
<td>12.82</td>
<td>12.40</td>
<td>12.00</td>
<td>11.68</td>
<td>11.63</td>
<td>11.97</td>
<td>13.24</td>
</tr>
<tr>
<td>Value of Operations</td>
<td>233.98</td>
<td>241.96</td>
<td>250.00</td>
<td>256.87</td>
<td>257.86</td>
<td>250.68</td>
<td>226.65</td>
</tr>
<tr>
<td>Value of Debt</td>
<td>0.00</td>
<td>24.20</td>
<td>50.00</td>
<td>77.06</td>
<td>103.14</td>
<td>125.34</td>
<td>135.99</td>
</tr>
<tr>
<td>Value of Equity</td>
<td>233.98</td>
<td>217.76</td>
<td>200.00</td>
<td>179.81</td>
<td>154.72</td>
<td>125.34</td>
<td>90.66</td>
</tr>
<tr>
<td># Shares</td>
<td>12.72</td>
<td>11.45</td>
<td>10.14</td>
<td>8.81</td>
<td>7.54</td>
<td>6.41</td>
<td>5.73</td>
</tr>
<tr>
<td>Shares Repurchased***</td>
<td></td>
<td>1.27</td>
<td>1.31</td>
<td>1.33</td>
<td>1.27</td>
<td>1.13</td>
<td>0.67</td>
</tr>
<tr>
<td>$$ Value per Share</td>
<td>18.40</td>
<td>19.03</td>
<td>19.73</td>
<td>20.41</td>
<td>20.52</td>
<td>19.57</td>
<td>15.81</td>
</tr>
<tr>
<td>Net Income</td>
<td>30.00</td>
<td>28.87</td>
<td>27.60</td>
<td>26.07</td>
<td>23.87</td>
<td>20.98</td>
<td>16.95</td>
</tr>
<tr>
<td>$$ EPS</td>
<td>2.36</td>
<td>2.52</td>
<td>2.72</td>
<td>2.96</td>
<td>3.17</td>
<td>3.27</td>
<td>2.96</td>
</tr>
</tbody>
</table>

Notes: *starting capital structure, **optimal capital structure, ***not presented in original example

Another likely student approach would be to start at the zero level of financial leverage and work through the increasing levels of financial leverage referring at each step back to the zero level data. This is similar to what is done in the original Brigham example, albeit with an anchor point at the .20 total debt ratio level of financial leverage. Again, the resulting values will be the same up through the number of shares outstanding. Thereafter, all of the values relating to shares outstanding or repurchased will be different from the base case of the Brigham example. The values resulting from this sequence of calculations is presented in Table 3. Note that in both Table 2 and Table 3, the variation in the number of share calculations does not change the optimal degree of financial leverage which yields the highest firm value.
Table 3: Variation Two – Anchored from Zero (Changes in Italics)

<table>
<thead>
<tr>
<th></th>
<th>Wd</th>
<th>0*</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4**</th>
<th>0.5</th>
<th>0.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ws</td>
<td>1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Pre-tax rd (%)</td>
<td>7.70</td>
<td>7.80</td>
<td>8.00</td>
<td>8.50</td>
<td>9.90</td>
<td>12.00</td>
<td>16.00</td>
<td></td>
</tr>
<tr>
<td>After-tax rd (%)</td>
<td>4.62</td>
<td>4.68</td>
<td>4.80</td>
<td>5.10</td>
<td>5.94</td>
<td>7.20</td>
<td>9.60</td>
<td></td>
</tr>
<tr>
<td>β</td>
<td>1.0870</td>
<td>1.1594</td>
<td>1.2500</td>
<td>1.3665</td>
<td>1.5217</td>
<td>1.7391</td>
<td>2.0652</td>
<td></td>
</tr>
<tr>
<td>rm (%)</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
<td>12.30</td>
</tr>
<tr>
<td>rf (%)</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td>6.30</td>
<td></td>
</tr>
<tr>
<td>rs (%)</td>
<td>12.82</td>
<td>13.26</td>
<td>13.80</td>
<td>14.50</td>
<td>15.43</td>
<td>16.73</td>
<td>18.69</td>
<td>18.69</td>
</tr>
<tr>
<td>WACC (%)</td>
<td>12.82</td>
<td>12.40</td>
<td>12.00</td>
<td>11.68</td>
<td>11.63</td>
<td>11.97</td>
<td>13.24</td>
<td>13.24</td>
</tr>
<tr>
<td>Value of Operations</td>
<td>233.98</td>
<td>241.96</td>
<td>250.00</td>
<td>256.87</td>
<td>257.86</td>
<td>250.68</td>
<td>226.65</td>
<td></td>
</tr>
<tr>
<td>Value of Debt</td>
<td>233.98</td>
<td>217.76</td>
<td>200.00</td>
<td>179.81</td>
<td>154.72</td>
<td>125.34</td>
<td>135.99</td>
<td>90.66</td>
</tr>
<tr>
<td>Value of Equity</td>
<td>233.98</td>
<td>241.96</td>
<td>250.00</td>
<td>256.87</td>
<td>257.86</td>
<td>250.68</td>
<td>226.65</td>
<td></td>
</tr>
<tr>
<td># Shares</td>
<td>12.72</td>
<td>11.45</td>
<td>10.17</td>
<td>8.90</td>
<td>7.63</td>
<td>6.36</td>
<td>5.09</td>
<td></td>
</tr>
<tr>
<td>Shares Repurchased***</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td>1.27</td>
<td></td>
</tr>
<tr>
<td>$ Value per Share</td>
<td>18.40</td>
<td>19.03</td>
<td>19.66</td>
<td>20.20</td>
<td>20.28</td>
<td>19.71</td>
<td>17.82</td>
<td></td>
</tr>
<tr>
<td>Net Income</td>
<td>30.00</td>
<td>28.87</td>
<td>27.60</td>
<td>26.07</td>
<td>23.87</td>
<td>20.98</td>
<td>16.95</td>
<td></td>
</tr>
<tr>
<td>$ EPS</td>
<td>2.36</td>
<td>2.52</td>
<td>2.71</td>
<td>2.93</td>
<td>3.13</td>
<td>3.30</td>
<td>3.33</td>
<td></td>
</tr>
</tbody>
</table>

Notes: *starting capital structure, **optimal capital structure, ***not presented in original example

Clearly, the objective of the numerical example is to provide the student with reinforcement. In this case a relatively minor design flaw in the example can lead to an unnecessary potential confusion for students. Instructors using textbooks from the Brigham group should be aware of this issue and guide students in advance. And, the authors in the Brigham group should consider modifying the example to avoid this pitfall.

Variations on the basic numerical example approach presented above can easily be developed to accentuate other issues in optimal capital structure policy formation. One of these is the mention in several textbooks of the relatively “flat” nature of the functions in the midranges of financial leverage. This is typically presented using a graph which shows the value of the firm and the firm’s WACC being functions which have a near zero slope in the 20-50% total debt ratio range. (See, for example, Brigham and Ehrhardt, p. 613).

It is straightforward to develop variations in the basic example which demonstrate this effect. The key is to select a combination of example inputs which yield the desired result. Examples can also be constructed to yield “smooth” curves of the sort generally depicted in the purely graphical textbook representations. Of course, unusual “kinked” function examples can also be developed.

In the author’s experience it is useful to provide students with multiple examples which result in somewhat different functional forms. This serves to both reinforce the theoretical relationships and make students accustomed to the variations which may be encountered in real world estimations. It is beneficial to assign different variants to subsets of students in a given course. Then, the results, when revealed, can serve to spark a reinforcing classroom discussion.

Table 4 presents an example developed by the author to replicate the “flat” curves described for both the WACC and value function in real world settings by many sources. In this particular set of examples, the author adds hypothetical bond ratings as an aid to student comprehension of the increase in debt yields which occurs at higher levels of financial leverage. Figure 1 presents the WACC and firm value functions resulting from this numerical example. Note that it is generally assumed that the firm in such examples will not have any nonoperating assets, so the value of operations is the same as the value of the firm. In this example, the complication of the number of shares is omitted so the emphasis is on the relationship between firm value and the degree of financial leverage. Note that there is a local maximum at a total debt
ratio of .40, so the inattentive student may not identify the actual maximum. However, as intended, there is little difference in value in the midrange value for this example.

**Table 4**: Author’s Example Showing “Flat” Value Function

<table>
<thead>
<tr>
<th>$W_d$</th>
<th>0*</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6**</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_s$</td>
<td>1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Pre-tax $r_t$ (%)</td>
<td>3.05</td>
<td>3.11</td>
<td>3.38</td>
<td>4.52</td>
<td>6.13</td>
<td>7.57</td>
<td>8.15</td>
<td>9.62</td>
<td>12.25</td>
<td>15.68</td>
</tr>
<tr>
<td>After-tax $r_d$ (%)</td>
<td>1.86</td>
<td>1.90</td>
<td>2.06</td>
<td>2.76</td>
<td>3.74</td>
<td>4.62</td>
<td>4.97</td>
<td>5.87</td>
<td>7.47</td>
<td>9.56</td>
</tr>
<tr>
<td>Hypothetical Bond Rating***</td>
<td>Aaa</td>
<td>Aa</td>
<td>A</td>
<td>Baa</td>
<td>Ba</td>
<td>B</td>
<td>Caa</td>
<td>Ca</td>
<td>C</td>
<td>C*</td>
</tr>
<tr>
<td>$\beta$</td>
<td><strong>1.3200</strong></td>
<td>1.4095</td>
<td>1.5213</td>
<td>1.6651</td>
<td>1.8568</td>
<td>2.1252</td>
<td>2.5278</td>
<td>3.1988</td>
<td>4.5408</td>
<td>8.5668</td>
</tr>
<tr>
<td>$r_m$ (%)</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
<td>11.50</td>
</tr>
<tr>
<td>$r_f$ (%)</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>$r_s$ (%)</td>
<td>14.17</td>
<td>14.92</td>
<td>15.85</td>
<td>17.05</td>
<td>18.65</td>
<td>20.90</td>
<td>24.26</td>
<td>29.86</td>
<td>41.07</td>
<td>74.68</td>
</tr>
</tbody>
</table>

Value of Operations | 444.54 | 462.66 | 481.11 | 493.55 | 496.52 | 493.86 | 496.62 | 482.18 | 443.94 | 391.87 |

**Notes**: *starting capital structure, **optimal capital structure, ***added to aid student understanding

**Figure 1**: WACC and Firm Value for “Flat” Example

A second numerical example is presented in Table 5. In this example the values are selected so as to yield the theoretically implied smoothly curved functional forms without any reversals. The resulting functions are depicted in Figure 2. In this case, the value of the firm increases up to the maximum value...
and then decreases. There are no reversals and the functions match the expectations of one who had only been presented with the typical representative graphs.

A third numerical example is presented in Table 6. The graphs resulting from this example are presented in Figure 3. In this case, the values in the example are selected to render an unexpected shape for the functions which include multiple “kinks”. Here again the student would need to pay careful attention in order to identify the actual maximum value capital structure.

Table 5: Author’s Example Showing “Smooth” Value Function

<table>
<thead>
<tr>
<th></th>
<th>Wd</th>
<th>Ws</th>
<th>Pre-tax rd (%)</th>
<th>After-tax rd (%)</th>
<th>Hypothetical Bond Rating***</th>
<th>β</th>
<th>rm (%)</th>
<th>rf (%)</th>
<th>rs (%)</th>
<th>WACC (%)</th>
<th>Value of Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0**</td>
<td>0.1</td>
<td>0.2</td>
<td>0.3</td>
<td>Aaa</td>
<td>1.100</td>
<td>12.00</td>
<td>3.05</td>
<td>12.90</td>
<td>12.90</td>
<td>581.62</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
<td>Aa</td>
<td>1.174</td>
<td>12.00</td>
<td>3.05</td>
<td>13.56</td>
<td>12.49</td>
<td>600.50</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4**</td>
<td>0.8</td>
<td>1.0</td>
<td>A</td>
<td>1.267</td>
<td>12.00</td>
<td>3.05</td>
<td>14.40</td>
<td>12.12</td>
<td>618.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>1.0</td>
<td>1.2</td>
<td>Baa</td>
<td>1.387</td>
<td>12.00</td>
<td>3.05</td>
<td>15.47</td>
<td>11.79</td>
<td>636.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.6</td>
<td>0.8</td>
<td>1.0</td>
<td>Ba</td>
<td>1.547</td>
<td>12.00</td>
<td>3.05</td>
<td>16.90</td>
<td>11.66</td>
<td>642.99</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.4**</td>
<td>0.6</td>
<td>0.8</td>
<td>B</td>
<td>1.771</td>
<td>12.00</td>
<td>3.05</td>
<td>18.90</td>
<td>11.61</td>
<td>645.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.7</td>
<td>0.9</td>
<td>1.0</td>
<td>Caa</td>
<td>2.106</td>
<td>12.00</td>
<td>3.05</td>
<td>21.90</td>
<td>11.74</td>
<td>638.61</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.8</td>
<td>0.7</td>
<td>1.0</td>
<td>Ca</td>
<td>2.665</td>
<td>12.00</td>
<td>3.05</td>
<td>26.91</td>
<td>12.18</td>
<td>615.76</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.9</td>
<td>0.6</td>
<td>1.0</td>
<td>C</td>
<td>3.784</td>
<td>12.00</td>
<td>3.05</td>
<td>36.91</td>
<td>13.36</td>
<td>561.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.1</td>
<td>0.8</td>
<td>1.0</td>
<td>C*</td>
<td>7.139</td>
<td>12.00</td>
<td>3.05</td>
<td>66.94</td>
<td>15.30</td>
<td>490.11</td>
</tr>
</tbody>
</table>

Notes: *starting capital structure, **optimal capital structure, ***added to aid student understanding

Figure 2: WACC and Firm Value for “Smooth” Example
Table 6: Author’s Example Showing “Kinked” Value Function

<table>
<thead>
<tr>
<th></th>
<th>0*</th>
<th>0.1</th>
<th>0.2</th>
<th>0.3</th>
<th>0.4</th>
<th>0.5</th>
<th>0.6**</th>
<th>0.7</th>
<th>0.8</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_d$</td>
<td>1</td>
<td>0.9</td>
<td>0.8</td>
<td>0.7</td>
<td>0.6</td>
<td>0.5</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
</tr>
<tr>
<td>Pre-tax $r_d$ (%)</td>
<td>3.75</td>
<td>3.81</td>
<td>4.08</td>
<td>4.52</td>
<td>6.56</td>
<td>6.93</td>
<td>7.15</td>
<td>9.62</td>
<td>11.62</td>
<td>15.68</td>
</tr>
<tr>
<td>After-tax $r_d$ (%)</td>
<td>2.25</td>
<td>2.29</td>
<td>2.45</td>
<td>2.71</td>
<td>3.94</td>
<td>4.16</td>
<td>4.29</td>
<td>5.77</td>
<td>6.97</td>
<td>9.41</td>
</tr>
<tr>
<td>Hypothetical Bond Rating***</td>
<td>Aaa</td>
<td>Aa</td>
<td>A</td>
<td>Baa</td>
<td>Ba</td>
<td>B</td>
<td>Caa</td>
<td>Ca</td>
<td>C</td>
<td>C*</td>
</tr>
<tr>
<td>$\beta$</td>
<td>1.1500</td>
<td>1.2267</td>
<td>1.3225</td>
<td>1.4457</td>
<td>1.6100</td>
<td>1.8400</td>
<td>2.1850</td>
<td>2.7600</td>
<td>3.9100</td>
<td>7.3600</td>
</tr>
<tr>
<td>$r_m$ (%)</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
<td>11.00</td>
</tr>
<tr>
<td>$r_d$ (%)</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
<td>3.15</td>
</tr>
<tr>
<td>$r_s$ (%)</td>
<td>12.18</td>
<td>12.78</td>
<td>13.53</td>
<td>14.50</td>
<td>15.79</td>
<td>17.59</td>
<td>20.30</td>
<td>24.82</td>
<td>33.84</td>
<td>60.93</td>
</tr>
<tr>
<td>WACC (%)</td>
<td>12.18</td>
<td>11.73</td>
<td>11.31</td>
<td>10.96</td>
<td>11.05</td>
<td>10.88</td>
<td>10.69</td>
<td>11.49</td>
<td>12.35</td>
<td>14.56</td>
</tr>
<tr>
<td>Value of Operations</td>
<td>698.01</td>
<td>724.64</td>
<td>751.22</td>
<td>775.35</td>
<td>769.40</td>
<td>781.54</td>
<td>794.77</td>
<td>740.08</td>
<td>688.47</td>
<td>583.80</td>
</tr>
</tbody>
</table>

Notes: *starting capital structure, **optimal capital structure, ***added to aid student understanding.

Figure 3: WACC and Firm Value for “Kinked” Example

![WACC and Firm Value Graph](image-url)
Summary and Conclusions

This paper has discussed the use of numerical examples of the estimation of optimal capital structure in reinforcing student learning. Potential difficulties in using one such example appearing in a popular group of finance textbooks were discussed. Then variations on the type of numerical examples were presented which can be used to sensitize students to variations in outcomes which are likely to be encountered in actual real world capital structure estimations. With a modest amount of effort the basic numerical examples can be expanded and varied to provide a richer experience for the students.

References


Obtaining Consistent Corporate Valuations

William A. Ogden, Pedro Sottile,1 and Srinivasan Sundaram2

Abstract

In an introductory finance course, students are introduced to the concept of equity valuation in the form of the constant growth dividend discount model. In advanced courses they are exposed to additional valuation approaches such as free cash flows, adjusted present value, residual earnings and economic profit models. Unfortunately students do not have the opportunity to go beyond the calculations and learn the basic assumptions of each model that will lead to consistent valuations regardless of which model they choose. In this paper, we value an example company using 6 models that lead to identical equity values.

Introduction

In a typical introductory finance course, we introduce discounted cash flow (DCF) methodologies. In general, instructors initially apply DCF methods in order to value bonds and then to apply them to stocks. For common stock valuation, the dividend discount model fits nicely into the introductory course due to its simplicity. Most intermediate level texts such as Brigham and Daves (2013) and Brealey, Myers and Allen (2013) include the dividend discount model (DDM) followed later by several corporate valuation models that either discount free cash flows in order to arrive at an enterprise value, or discount cash flows to shareholders to value the company’s equity. Overall, we find that undergraduate students can understand the reasoning behind discounted cash flow methodologies but struggle to identify the key assumptions that are necessary such that all valuation models will lead to identical equity values.

Our purpose in this paper is to provide the basics of valuation models such that a finance professor can assign this paper as a reading and readily expect that intermediate-level students will be able to understand and replicate it with other example companies. We present a comprehensive example that demonstrates the equivalence of six valuation models, i.e., free cash flow (FCF), economic profit (EP), adjusted present value (APV), flow-to-equity (FTE), dividend discount (DDM) and residual earnings (RE) models. Rather than demonstrating how different models can lead to different valuations, we have consistent assumptions across all models resulting in all models having consistent and identical outcomes.

Fernandez (2013) provides a compendium of valuation methods. He values a company by applying 10 different models and 9 theories. While Fernandez is thorough, he does not provide a level of detailed calculations that we believe would be instructive at the undergraduate level. We find that most undergraduates encounter significant difficulties while attempting to follow his calculations. Accordingly we provide in-depth details in our calculations that will enable undergraduate students to learn the basics of valuation models. We build upon the valuation methods found in Koller, Goehart and Wessels (2010). They value a company by applying free cash flow (FCF), economic profit (EP) and adjusted present value (APV) valuations of a company’s equity. The three models independently estimate the same value. In appendices, Koller et. al. also demonstrate the algebraic equivalence of equations for APV, EP, and FCF. For our exercise, we include a larger variety models that students are likely to encounter in their upper level courses (or eventually to prepare for the CFA exam). Our models include EP and RE that begin with the book values and add to them by discounting cash flows; FCF and APV that solve directly for enterprise value; and DDM and FTE that solve directly for equity value. In addition, APV separates the after tax operating profits and the

1 William A. Ogden, Professor of Finance (bogden@uwec.edu), and Pedro Sottile, Assistant Professor of Finance (sottilpd@uwec.edu), both at College of Business, University of Wisconsin, Eau Claire, Eau Claire WI 54702-4004.

2 Srinivasan Sundaram, Professor of Finance (sundaram@bsu.edu), Miller College of Business, Ball State University, Muncie IN 47306.
interest tax shields and discounts them to find the enterprise value. Understanding the basics of valuing tax shields (in order to arrive at valuations that are consistent with the other models) will provide a foundation for studying the numerous other treatments of interest tax shields (that do not always lead to consistent valuations with the other models).

Obtaining consistent valuations among models requires basic assumptions. In the next section we describe them.

Assumptions

Consistent valuations result from consistent assumptions. We use the following set of assumptions across all valuation models:

1. Capital structure is rebalanced to maintain a constant debt/value ratio based on market values. Papaioannou (2014) shows the calculations that are necessary for consistency among valuation models for a company that maintains a constant debt value rather than a constant debt/value ratio3.

2. The company pays a dividend that is equal to its net income plus the change in debt less its investment in net operating working capital and operating long-term assets. In our example, the change in debt is the amount the company must either issue or retire in order to maintain a constant debt-to-value ratio. Operating working capital refers to operating current assets net of operating current liabilities. Operating assets refer to those real assets that pertain solely to operations and creating wealth.

3. The company’s debt sells at par. Fernandez (2013) and Papaioannou (2014) provide adjustments to basic models if debt does not sell at par. These adjustments are not necessary to accomplish our purpose.

Valuation Models

The following provides a brief description of the six valuation models that we will use:

Free Cash Flow (FCF)

The FCF model finds the enterprise value by discounting the unlevered free cash flows at the company’s weighted average cost of capital (WACC). Free cash flow is the difference between after-tax operating income and the investments made in net operating working capital and net fixed assets. Net operating working capital is essentially the difference between current assets that do not bear interest and current liabilities that do not bear interest.

\[
FCF = EBIT(1-T) - \Delta Net\ Operating\ Capital, \text{ where } \ \Delta Net\ Operating\ Capital = \Delta Net\ operating\ working\ capital + \Delta Net\ Fixed\ Assets.
\]

Alternatively, free cash flow can be calculated as:

\[
FCF = EBIT(1-T) + \text{Depreciation} - \Delta Net\ operating\ working\ capital - \Delta Gross\ Fixed\ Assets.
\]

Economic Profit (EP)

The EP model begins with the value of the company’s investor capital and adds to it the present value (discounted at the company’s WACC) of the company’s future expected economic profits. Investor capital is the book value of the company’s debt (short-term and long-term debt) and equity. Economic profit is the difference between after-tax operating profits and the capital charge. Investor capital can also be found from the left side of the balance sheet as the sum of net fixed assets and net operating working capital. The left side investor capital is also referred to as operating assets. Thus, we have an identity: Operating Capital = Investor Capital. The capital charge is assessed against the company’s after-tax operating profits and represents the cost to the company of using its investors’ capital for the past year. The difference between the after-tax operating profits and the capital charge is the economic profit.

\[
EP = EBIT(1-T) - \text{Capital Charge}
\]

---

3 Papaioannou (2014) highlights the robustness of the APV method and the importance of rebalancing a company’s capital structure as it relates to estimating the cost of capital.
Capital Charge = Beginning Investor Capital X WACC.

**Adjusted Present Value (APV)**

The APV method separates the operating cash flows and the interest tax shields. To arrive at enterprise value, the APV model adds the present value of the company’s free cash flows and the present value of the interest tax shields, both discounted at the cost of unlevered equity since the company rebalances in order to maintain a constant debt/value ratio based on market values. The unlevered free cash flows and the interest tax shields are equally risky and accordingly, we discount them at the same rate. Brealey, Myers, and Allen (2013) discount the unlevered cash flows at the cost of unlevered capital and the interest tax shields at the cost of debt for companies that do not maintain a constant debt/value ratio. Without constant rebalancing and discounting the interest tax shields at the cost of unlevered capital, the resulting APV valuation will not be consistent with the other models in this paper. The enterprise value is found as:

\[ V (APV) = PV \text{ unlevered free cash flows} + PV \text{ interest tax shields}, \]

For both terms, the discount rate \( r = \text{cost of unlevered capital} \)

**Flow to Equity (FTE)**

The flow to equity (FTE) model values the portion of the unlevered cash flows that is available to the equity shareholder. The FTE model calculates the present value of what remains of the company’s free cash flows after adjusting for changes in debt and deducting interest (on an after-tax basis). The flows to equity are discounted at the cost of equity.

\[ \text{FTE} = \text{FCF} + \Delta \text{Debt} - \text{interest} + \text{interest tax shield}. \]

**Dividend Discount Model (DDM)**

The dividend discount model (DDM) is typically presented in an introductory finance course as a model for stock valuation. Stock value is found as the present value of the company’s expected dividends discounted at the cost of equity. The dividend policy that provides valuations consistent with the other models is:

\[ \text{Dividend} = \text{Net Income} + \Delta \text{Debt} - \Delta \text{Operating Assets}. \]

**Residual Earnings (RE)**

The residual earnings (RE) model assesses a capital charge against the company’s after-tax earnings. The cost of using shareholders’ capital for a year we call the equity capital charge. Residual earnings are the difference between book earnings and the equity capital charge. The RE model adds beginning book equity plus the present value of residual earnings (discounted at the cost of equity) to obtain the market value of equity.

\[ \text{RE} = \text{Net Income} - \text{Equity Capital Charge} \]
\[ \text{Equity Capital Charge} = \text{Cost of Equity} \times \text{Beginning Book Equity} \]

The above models provide sufficient variety of valuation approaches that a student can learn the relationships among them while becoming familiar enough with them to understand the effects of altering the basic assumptions that we make. As an example, Myers (1974) discounts interest tax shields at the company’s cost of debt (appropriate for a company that does not rebalance to maintain a constant Debt/Value ratio); Fernandez (2007) calculates the interest tax shield by multiplying the unlevered cost of capital times the value of the company’s debt times the corporate tax rate. He then discounts the tax shields at the unlevered cost of capital. Ruback (2002), Papaioannou (2014) and Fernandez (2013) present valuations that require that the cost of equity and the weighted average cost of capital be recalculated each year and cash flows be discounted using multiple discount rates each period beyond one year out.

We value an example company by applying each of the above valuation models and arrive at the same equity value for all models. In the following section, we describe the operating characteristics of the example company.
The S. Walker Company Example

S. Walker is a manufacturing company in northern Indiana that had sales of $85 million this year. The sales forecast is 7% growth for the next three years and 4% thereafter. The company’s operating forecasts are based on the following ratios:

- Cost of Goods Sold/Sales: 75%
- Net Operating Working Capital/Sales: 15%
- Net fixed assets/Sales: 65%
- Depreciation/Net Fixed Assets: 12%

The company uses the following values for its cost of capital calculations:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate on debt</td>
<td>( r_D )</td>
</tr>
<tr>
<td>Tax rate</td>
<td>( T )</td>
</tr>
<tr>
<td>Leverage ratio</td>
<td>( L ) (debt/value)</td>
</tr>
<tr>
<td>Unlevered cost of capital</td>
<td>( r_U )</td>
</tr>
</tbody>
</table>

S. Walker has 1.5 million shares of equity outstanding. Based on the given operating ratios, we estimate unlevered cash flow for the next 4 years. We commonly refer to these as unlevered free cash flows or free cash flows. They are shown in Panel A of Table 1.

Net operating working capital is operating current assets net of operating current liabilities. “Operating” indicates that they are connected directly to operations (not short-term investments or short-term notes). Net operating assets are the sum of net operating working capital and net fixed assets. Investment in fixed assets is defined as the difference between ending and beginning net fixed assets plus depreciation. For year 1, it is calculated as:

\[
\text{Investment in fixed assets} = \text{ending net fixed assets} - \text{beginning net fixed assets} + \text{depreciation}
\]

The unlevered free cash flows (or free cash flows) are independent of the company’s financial leverage. Hence, we do not include interest expense in the calculation. Free Cash Flow (FCF) calculations begin with net operating profits after taxes (NOPAT), add back non cash operating expenses (typically depreciation and amortization) and subtract investments in net operating capital. For year 1, NOPAT and free cash flow equal:

\[
\text{NOPAT (1)} = \text{EBIT} - \text{operating taxes} = \text{EBIT}(1-T) = 15.64 - 5.48 = 10.17
\]

\[
\text{FCF} = \text{NOPAT} + \text{Depreciation} - (\text{Increase in NOWC}) - (\text{Increase in Fixed Assets})
\]

\[
\text{FCF (1)} = 10.17 + 7.09 - (13.64 - 12.75) - (59.12 - 55.25 + 7.09)= 5.41
\]

### Table 1. Investor Capital and Free Cash Flows

<table>
<thead>
<tr>
<th>Panel A. Investor Capital</th>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Net operating working capital</td>
<td>12.75</td>
<td>13.64</td>
<td>14.60</td>
<td>15.62</td>
<td>16.24</td>
<td>(15% Sales)</td>
</tr>
<tr>
<td>Net fixed assets</td>
<td>55.25</td>
<td>59.12</td>
<td>63.26</td>
<td>67.68</td>
<td>70.39</td>
<td>(65% Sales)</td>
</tr>
<tr>
<td>Depreciation</td>
<td>6.63</td>
<td>7.09</td>
<td>7.59</td>
<td>8.12</td>
<td>8.45</td>
<td>(12% NFA)</td>
</tr>
<tr>
<td>Net Operating Assets</td>
<td>68.00</td>
<td>72.76</td>
<td>77.85</td>
<td>83.30</td>
<td>86.64</td>
<td>(NOWC + NFA)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B Free Cash Flows</th>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>85.00</td>
<td>90.95</td>
<td>97.32</td>
<td>104.13</td>
<td>108.29</td>
<td>(75% X Sales)</td>
</tr>
<tr>
<td>Operating Costs</td>
<td>68.21</td>
<td>72.99</td>
<td>78.10</td>
<td>81.22</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBITDA</td>
<td>22.74</td>
<td>24.33</td>
<td>26.03</td>
<td>27.07</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>7.09</td>
<td>7.59</td>
<td>8.12</td>
<td>8.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>EBIT</td>
<td>15.64</td>
<td>16.74</td>
<td>17.91</td>
<td>18.63</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Taxes</td>
<td>5.48</td>
<td>5.86</td>
<td>6.27</td>
<td>6.52</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NOPAT</td>
<td>10.17</td>
<td>10.88</td>
<td>11.64</td>
<td>12.11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>7.09</td>
<td>7.59</td>
<td>8.12</td>
<td>8.45</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Operating cash flow</td>
<td>17.26</td>
<td>18.47</td>
<td>19.76</td>
<td>20.55</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Increase in NOWC</td>
<td>0.89</td>
<td>0.95</td>
<td>1.02</td>
<td>0.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Investment in fixed assets</td>
<td>10.96</td>
<td>11.73</td>
<td>12.55</td>
<td>11.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unlevered free cash flows</td>
<td>5.41</td>
<td>5.79</td>
<td>6.19</td>
<td>8.78</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
We complete the FCF calculations for the next 3 years using the same equations that we show above. Having calculated the unlevered free cash flows, we will complete our first valuation using the free cash flow model.

**Free Cash Flow (FCF) Valuation**

The FCF model finds the enterprise value by discounting the unlevered free cash flows at the company’s weighted average cost of capital (WACC). To find the WACC, we first find the cost of levered equity ($r_E$). Later, $r_E$ will also serve as the discount rate for the dividend discount, the residual earnings and the flow to equity models. At this point, we commit the company to maintaining a constant leverage ratio (Debt/Value) based on market values of 0.20. Maintaining a constant debt ratio leads to all models estimating identical equity values for the company. We employ the cost of equity equation suggested by Harris and Pringle (1985), Brigham and Daves (2013) and Brealey, Myers, and Allen (2013) among others:

$$r_E = r_U + (r_U - r_D)(D/E) = 9.840\% - (9.840\% - 6\%)(0.20/0.80) = 10.80\%$$

With the debt ratio (debt/value) of 20%, a tax rate of 35%, a cost of debt of 6% and the cost of equity of 10.80%, the weighted average cost of capital (WACC) equals:

$$WACC = D/V \times r_D (1-T) + E/V \times r_E = 20\% (6.0\%)(1-35\%) + 80\% (10.80\%) = 9.42\%.$$  

We have estimated that the S. Walker Company will grow at a constant rate of 4% after year 3. To find the value of free cash flows that occur after year 3, we calculate the company’s continuing value (also referred to as the horizon value) in year 3 as:

$$CV(3) = \frac{FCF_4}{WACC - g} = 8.78 / (9.42\% - 4\%) = 161.90$$

As shown in Panel C of Table 1, we add the continuing value to the year 3 free cash flow to obtain a total year 3 free cash flow of 161.90 + 6.19 = 168.09. Discounting the total free cash flows at the WACC results in an Enterprise Value = Debt + Equity (at market values) of 138.09. Given a debt/value ratio of 20%, the free cash flows will support an initial debt level of:

$$Debt(0) = 20\% (138.09) = 27.62.$$

Subtracting debt from the enterprise value results in an equity value of:

$$138.09 - 27.62 = 110.47,$$  

or 110.47 / 1.5 (shares) = 73.65 per share.

In order to value the company with the other 5 models, we need to find the market value of the company and its debt for the following 4 years. We present these values in Panel D of Table 1.

Value is found by discounting the future free cash flows at the WACC as follows:

Value (1) = [PV($r = 9.42\%$, 5.79, (6.19+161.90))] = 145.69  

Value (2) = [PV($r = 9.42\%$, (6.19+161.90))] = 153.62  

Value (3) = CV(3) = 161.90 (the continuing value)  

Value (4) = Value (3) (1+g) = 161.90 (1.04) = 168.38.

Interest tax shields are the product of interest and the tax rate. For year 1, interest is:

$$Interest(1) = \text{beginning debt} \times \text{before-tax interest rate} = 27.62 \times 6\% = 1.66$$

The resulting interest tax shield is:
Interest tax shield (1) = interest × tax rate = 1.66 (35%) = 0.58

Each year, the company maintains a debt/value ratio equal to 20% of market value. Rebalancing is essential in order to produce consistent valuations across all models. In the next section, we present an Economic Profit (EP) valuation.

**Economic Profit (EP) Valuation**

The EP model begins with the value of the company’s investor funds and adds to it the present value (discounted at the company’s WACC) of the company’s future expected economic profits. Economic profit is the difference between after-tax operating profits and the capital charge. The capital charge is assessed against the company’s after-tax operating profits and represents the cost to the company of using its investors’ capital for the past year. The difference between the after-tax operating profits and the capital charge is the economic profit. The capital charge emphasizes the importance of capital being productive. If investor capital is employed unproductively, it generates relatively low economic profits. Knowing that the capital charge will be assessed against after-tax operating profits deters managers from investing large amounts of capital in order to generate higher levels of book earnings (at the cost of high capital charges and the resulting low economic profits).

The EP model is one of two presented in this paper that begins with the book value of the employed capital. The other is the residual earnings model which begins with the book value of equity. From Panel A of Table 2, we have employed initial values of net operating working capital (NOWC) and net fixed assets (NFA) of 12.75 and 55.25, respectively for a total net operating assets of 68.00. The unlevered free cash flows are sufficient to support an initial debt level of 27.62 (from Panel D of Table 1). We use the book value of debt as an estimate of its market value. Net operating assets are equal to invested capital. We find the book value of the company’s equity by subtracting the initial debt from the initial invested capital, resulting in a “book” equity amount of 68.00 – 27.62 = 40.38. The EP valuation is presented in Panel B of Table 2. For year 1, the company had employed 68.00 of investor capital. The capital charge for the use of that capital for year 1 is:

\[
\text{Capital charge (1)} = \text{Invested capital (T = 0)} \times \text{WACC} = 68.00 \times (9.42\%) = 6.41.
\]

Subtracting the capital charge from the year 1 NOPAT results in an economic profit of:

\[
\text{EP(1)} = \text{NOPAT}_1 - \text{Capital charge}_1 = 10.17 - 6.41 = 3.76.
\]

For year 3, the continuing value is:

\[
\text{CV(3)} = \frac{\text{EP}_4}{(\text{WACC} - g)} = \frac{4.26}{(9.42\% - 4\%)} = 78.60.
\]

We add the present value of the economic profits (70.09) to the beginning investor capital (68.00) to arrive at the enterprise value of 138.09, which is the precise value that we found by discounting the free cash flows at the WACC. As with the FCF method, we subtract the T= 0 debt value (27.62) to find the equity value of 110.47.

In the following section, we present the Adjusted Present Value model, which also solves for enterprise value but discounts the unlevered cash flows and the interest tax shields at the unlevered cost of capital (\(r_U\)).

**Adjusted Present Value (APV)**

The APV method separates the operating cash flows and the interest tax shields. Most commonly the unlevered cash flows are discounted at the cost of unlevered capital and the tax shields at the cost of debt (Brealey, Myers, and Allen 2013). However, in our example we maintain a constant debt / value ratio based on market values. Debt is tied directly to the enterprise value, which is determined by the unlevered cash flows. Thus, the interest tax shields are equally risky as the unlevered cash flows and are also discounted at the unlevered cost of capital. If the company does not maintain a constant debt / value ratio, several discount rates have been suggested for the interest tax shields. As an example, Myers (1974) argues that the tax shields are equally risky as the debt itself and should therefore be discounted at the cost of debt. Very similar to APV is the capital cash flow model (CCF) from Ruback (2002). Capital cash flows are equivalent to the sum of the unlevered free cash flows and the interest tax shields; in the CCF model, the capital cash flows are discounted at the unlevered cost of capital (regardless of the debt/value ratio). Here, we will discount the unlevered cash flows and the tax shields separately, which is the conventional APV approach.

The APV begins by calculating the “base case” value. This is the enterprise value if the company was financed completely with equity. We find the base case value by discounting the unlevered cash flows...
at the unlevered cost of capital ($r_U$). In order to estimate the enterprise value of a levered company, we add the present value of the interest tax shields (in our example, also discounted at the cost of unlevered capital) to the base case value.

Panel C of Table 2 presents the APV valuation of the S. Walker Company. The continuing value in year 3 is similar to that of the FCF model except the discount rate is the unlevered cost of capital ($r_U$) rather than the WACC:

\[
CV(3) = \frac{FCF_4}{(r_U - g)} = \frac{8.78}{(9.84\% - 4\%)} = 150.26.
\]

Similarly, the continuing value for the interest tax shields is:

\[
CV_3 = \frac{\text{Interest tax shield}_4}{(r_U - g)} = \frac{0.68}{(9.84\% - 4\%)} = 11.64
\]

The enterprise value is the sum of the base case value and the present value of the interest tax shields. We find the equity value by subtracting the initial debt value from the enterprise value resulting in an equity value of 110.47, again equaling what we found using the other models.

### Table 2. Economic Profit and Adjusted Present Value Models

<table>
<thead>
<tr>
<th>Year</th>
<th>Panel A. Initial Balance Sheet (book values)</th>
<th>Panel B. Economic Profit Valuation</th>
<th>Panel C. Adjusted Present Value (APV) Valuation</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Net Operating Working Capital</td>
<td>NOPAT</td>
<td>Unlevered cash flows (from Table 1)</td>
</tr>
<tr>
<td>0</td>
<td>12.75</td>
<td>10.17</td>
<td>5.41</td>
</tr>
<tr>
<td>1</td>
<td>13.64</td>
<td>10.88</td>
<td>5.79</td>
</tr>
<tr>
<td>2</td>
<td>14.60</td>
<td>11.64</td>
<td>6.19</td>
</tr>
<tr>
<td>3</td>
<td>15.62</td>
<td>12.11</td>
<td>8.78</td>
</tr>
<tr>
<td>4</td>
<td>16.24</td>
<td></td>
<td>150.26</td>
</tr>
<tr>
<td></td>
<td>Net Fixed Assets</td>
<td>Net Operating Capital</td>
<td>Total unlevered cash flows</td>
</tr>
<tr>
<td>0</td>
<td>55.25</td>
<td>68.00</td>
<td>5.41</td>
</tr>
<tr>
<td>1</td>
<td>59.12</td>
<td>72.76</td>
<td>5.79</td>
</tr>
<tr>
<td>2</td>
<td>63.26</td>
<td>77.85</td>
<td>6.19</td>
</tr>
<tr>
<td>3</td>
<td>67.68</td>
<td>83.30</td>
<td>8.78</td>
</tr>
<tr>
<td>4</td>
<td>70.39</td>
<td>86.64</td>
<td>156.45</td>
</tr>
<tr>
<td></td>
<td>Investor Capital</td>
<td>Capital charge</td>
<td>PV, $r = r_U = 9.84%</td>
</tr>
<tr>
<td>0</td>
<td>68.00</td>
<td>6.41</td>
<td>127.78</td>
</tr>
<tr>
<td>1</td>
<td>72.76</td>
<td>6.85</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>77.85</td>
<td>7.33</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>83.30</td>
<td>7.85</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>86.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Debt</td>
<td>Economic profit</td>
<td>Tax shields (from Table 2)</td>
</tr>
<tr>
<td>0</td>
<td>27.62</td>
<td>3.76</td>
<td>0.58</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td>4.03</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td>4.31</td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td>4.26</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Equity</td>
<td>Continuing value</td>
<td>Continuing Value</td>
</tr>
<tr>
<td>0</td>
<td>40.38</td>
<td>3.76</td>
<td>0.58</td>
</tr>
<tr>
<td>1</td>
<td>43.62</td>
<td>4.03</td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>47.13</td>
<td>4.31</td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>50.92</td>
<td>4.26</td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>52.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Investor Capital</td>
<td>Total economic profit</td>
<td>Total tax shields</td>
</tr>
<tr>
<td>0</td>
<td>68.00</td>
<td>78.60</td>
<td>0.58</td>
</tr>
<tr>
<td>1</td>
<td>72.76</td>
<td></td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>77.85</td>
<td></td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>83.30</td>
<td></td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>86.64</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Enterprise value</td>
<td></td>
<td>Total tax shields</td>
</tr>
<tr>
<td>0</td>
<td>40.38</td>
<td></td>
<td>0.58</td>
</tr>
<tr>
<td>1</td>
<td>43.62</td>
<td></td>
<td>0.61</td>
</tr>
<tr>
<td>2</td>
<td>47.13</td>
<td></td>
<td>0.65</td>
</tr>
<tr>
<td>3</td>
<td>50.92</td>
<td></td>
<td>0.68</td>
</tr>
<tr>
<td>4</td>
<td>52.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Debt</td>
<td></td>
<td>Total value</td>
</tr>
<tr>
<td>0</td>
<td>27.62</td>
<td></td>
<td>138.09</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>27.62</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>110.47</td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td>1.50</td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td>73.65</td>
</tr>
<tr>
<td></td>
<td>Equity</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Shares</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Value / share</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Up to this point, we have presented valuation models in which we find the enterprise value and subtract the debt value to arrive at the equity value. In the next section, we present the flow to equity model.
– the first of three valuation models that directly estimates equity value without first estimating enterprise value.

**Flow-to-Equity**

The flow-to-equity (FTE) model values the portion of the unlevered cash flows that is available to the equity shareholder. The FTE valuation is shown in Panel A of Table 3. The calculation for flow to equity is:

\[
FTE = \text{Unlevered cash flow} + \text{change in debt} - \text{interest} + \text{interest tax shield.}
\]

\[
FTE (1) = 5.41 + 1.52 - 1.66 + 0.58 = 5.85.
\]

The unlevered cash flows, the changes in debt, interest, and interest tax shields are taken from Panels B and D of Table 1. The continuing value calculation is:

\[
CV (3) = \frac{FTE_4}{(r_E - g)} = \frac{8.81}{(10.80\% - 4\%)} = 129.52.
\]

We discount the FTE cash flows at the cost of levered equity \((r_E)\) of 10.80\% (calculated previously as a component cost of the WACC). The FTE model equity value of 73.65 is identical to equity valuations from the other models. Similarly to FTE, the next two models, the dividend discount and equity residual models also directly estimate equity value.

**Dividend Discount Model**

The dividend discount model (DDM) is typically presented in an introductory finance course. Here we will demonstrate that with the appropriate dividend payout policy, DDM will arrive at the exact equity value as the other models. The DDM valuation is presented in Panel B of Table 3. In order to arrive at a valuation that is consistent with our other models, we specify the dividend policy as:

\[
\text{Dividend} = \text{Earnings after taxes} + \text{change in debt} - \text{invested capital.}
\]

\[
\text{Dividend (1)} = 9.09 + (29.14 - 27.62) - (72.76 - 68.00) = 5.85.
\]

After estimating the dividends, we discount them at the cost of levered equity \((r_E)\). Alternatively, we can estimate the dividend by subtracting the change in book equity from the after-tax earnings. This is referred to as “clean surplus” relation (Pinto, Henry, Robinson, and Stowe 2010). Between year 0 and year 1, book equity increased by 43.62 – 40.38 = 3.24. This leads to a dividend of:

\[
\text{Dividend (1)} = 9.09 - 3.24 = 5.85.
\]

The DDM continuing value is calculated as:

\[
CV (3) = \frac{\text{Dividend}_4}{(r_E - g)} = \frac{8.81}{(10.80\% - 4\%)} = 129.52.
\]

Dividends are discounted at the cost of levered equity \((r_E)\). The equity value is 110.47, as it has been for all models. By no coincidence, the dividends are equal to the flows to equity from Panel A. They will be equal as long as the company pays out a dividend that is consistent with maintaining a clean surplus relationship between earnings and dividends and rebalancing debt to maintain a constant debt/value ratio. In the following section, we present our final model that estimates equity value directly - the residual earnings model.

**Residual Earnings**

The economic profit model assessed a capital charge against the company’s net after-tax operating profit. The residual earnings model is similar except it assesses a capital charge against the company’s after-tax earnings rather than against the net after-tax operating profit. We call the cost of using equity shareholders’ capital for a year the equity capital charge. Panel C of Table 3 presents the residual earnings valuation of the S. Walker Company. We calculate the equity capital charge for year 1 as follows:
Table 3. Direct Equity Valuations

### Panel A. Flow to Equity Valuation

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unlevered cash flows</td>
<td>5.41</td>
<td>5.79</td>
<td>6.19</td>
<td>8.78</td>
<td></td>
</tr>
<tr>
<td>Change in debt</td>
<td>1.52</td>
<td>1.59</td>
<td>1.66</td>
<td>1.30</td>
<td></td>
</tr>
<tr>
<td>Interest</td>
<td>1.66</td>
<td>1.75</td>
<td>1.84</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>Tax shields</td>
<td>0.58</td>
<td>0.61</td>
<td>0.65</td>
<td>0.68</td>
<td></td>
</tr>
<tr>
<td>Flow to equity (FTE)</td>
<td>5.85</td>
<td>6.24</td>
<td>6.65</td>
<td>8.81</td>
<td></td>
</tr>
<tr>
<td>Continuing value</td>
<td>129.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total FTE</td>
<td>5.85</td>
<td>6.24</td>
<td>136.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV, FTE, ( r = r_E = 10.80% )</td>
<td>110.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shares</td>
<td>1.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value / share</td>
<td>73.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel B. Dividend Discount Model

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sales</td>
<td>85.00</td>
<td>90.95</td>
<td>97.32</td>
<td>104.13</td>
<td>108.29</td>
</tr>
<tr>
<td>Operating Costs</td>
<td>68.21</td>
<td>72.99</td>
<td>78.10</td>
<td>81.22</td>
<td></td>
</tr>
<tr>
<td>EBITDA</td>
<td>22.74</td>
<td>24.33</td>
<td>26.03</td>
<td>27.07</td>
<td></td>
</tr>
<tr>
<td>Depreciation</td>
<td>7.09</td>
<td>7.59</td>
<td>8.12</td>
<td>8.45</td>
<td></td>
</tr>
<tr>
<td>EBIT</td>
<td>15.64</td>
<td>16.74</td>
<td>17.91</td>
<td>18.63</td>
<td></td>
</tr>
<tr>
<td>Interest (from Table 1, Panel C)</td>
<td>1.66</td>
<td>1.75</td>
<td>1.84</td>
<td>1.94</td>
<td></td>
</tr>
<tr>
<td>Taxes (35%)</td>
<td>4.90</td>
<td>5.25</td>
<td>5.62</td>
<td>5.84</td>
<td></td>
</tr>
<tr>
<td>Earnings after taxes</td>
<td>9.09</td>
<td>9.74</td>
<td>10.44</td>
<td>10.84</td>
<td></td>
</tr>
<tr>
<td>Invested capital</td>
<td>68.00</td>
<td>72.76</td>
<td>77.85</td>
<td>83.30</td>
<td>86.64</td>
</tr>
<tr>
<td>Book Value Debt</td>
<td>27.62</td>
<td>29.14</td>
<td>30.72</td>
<td>32.38</td>
<td>33.68</td>
</tr>
<tr>
<td>Book Value Equity</td>
<td>40.38</td>
<td>43.62</td>
<td>47.13</td>
<td>50.92</td>
<td>52.96</td>
</tr>
<tr>
<td>Increase in book equity</td>
<td>3.24</td>
<td>3.51</td>
<td>3.79</td>
<td>2.04</td>
<td></td>
</tr>
<tr>
<td>dividends</td>
<td>5.85</td>
<td>6.24</td>
<td>6.65</td>
<td>8.81</td>
<td></td>
</tr>
<tr>
<td>Continuing value</td>
<td>129.52</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total dividend cash flows</td>
<td>5.85</td>
<td>6.24</td>
<td>136.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV dividends, ( r = r_E = 10.80% )</td>
<td>110.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shares</td>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value / share</td>
<td>73.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Panel C. Residual Earnings Valuation

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Earnings after tax (from Panel B)</td>
<td>9.09</td>
<td>9.74</td>
<td>10.44</td>
<td>10.84</td>
<td></td>
</tr>
<tr>
<td>Equity at book value</td>
<td>40.38</td>
<td>43.62</td>
<td>47.13</td>
<td>50.92</td>
<td>52.96</td>
</tr>
<tr>
<td>Equity capital charge</td>
<td>4.36</td>
<td>4.71</td>
<td>5.09</td>
<td>5.50</td>
<td></td>
</tr>
<tr>
<td>Dividend</td>
<td>5.85</td>
<td>6.24</td>
<td>6.65</td>
<td>8.81</td>
<td></td>
</tr>
<tr>
<td>Addition to retained earnings</td>
<td>3.24</td>
<td>3.51</td>
<td>3.79</td>
<td>2.04</td>
<td></td>
</tr>
<tr>
<td>Residual earnings</td>
<td>4.73</td>
<td>5.03</td>
<td>5.35</td>
<td>5.34</td>
<td></td>
</tr>
<tr>
<td>Continuing value</td>
<td>78.60</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total residual earnings</td>
<td>4.73</td>
<td>5.03</td>
<td>83.95</td>
<td></td>
<td></td>
</tr>
<tr>
<td>PV, residual earnings, ( r = r_E = 10.80% )</td>
<td>70.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equity value</td>
<td>110.47</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shares</td>
<td>1.50</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Value / share</td>
<td>73.65</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Equity Capital Charge = Beginning book equity \( \times \) Cost of levered equity.

Equity Capital Charge (1) = 40.38 \( \times \) 10.80\% = 4.36.

Residual earnings are the difference between net income and the equity capital charge. For year 1, we calculate residual earnings (RE) as follows:

\[
RE (1) = Earnings \text{ after taxes} - \text{Capital charge} = 9.09 - 4.36 = 4.73
\]

We calculate the year 3 continuing value as follows:

\[
CV (3) = \frac{RE_3}{(r_E - g)} = \frac{8.81}{(10.80\% - 4\%)} = 78.60.
\]
After calculating each year’s total residual earnings, we discount them at the cost of levered equity (rE) to find the additional equity value that the company will create beginning in year 1. To find equity value at year 0, we add beginning book equity (40.38) and the present value of the residual earnings (70.09) to arrive at an equity value of 110.47 and a share value of 73.65.

**Conclusion**

For undergraduate students, learning and becoming proficient with corporate valuation models can be challenging. We value an example company with 6 models that students can expect to see again in their advanced finance courses and in their graduate programs. With the assumptions that we specify, students can be confident that they are applying the models correctly if the resulting equity values from all models are equal. We can assure that our students are proficient with the models by assigning exercises that are similar but different enough to prevent mechanical replication of our example. One such variation is to initially provide a levered beta rather than the unlevered cost of capital. Numerous other variations are possible. Once students are familiar with the basics of the models, they will be in a stronger position to begin changing our initial assumptions and making appropriate adjustments.

**References**


