Professor Gump's Dilemma: A Classroom Exercise in Game Theory

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ABSTRACT

Consider the dilemma of a professor who has handed back an exam to his students, but has forgotten to record their grades – can he develop a scheme that will provide each student the incentive to report his grade truthfully? This situation is used as an in-class exercise which develops the students' understanding of a Nash equilibrium and of how to use game theory to solve interesting everyday problems.

Introduction

Game theory has opened many doors for economic analysis with its focus on the strategic behavior of the players and is now a standard part of the undergraduate economics curriculum (Trandel 1999). The majority of introductory economics textbooks use game theory to offer an explanation to interesting questions by providing a well-defined set of players, rules, strategies, and payoff matrices -- e.g. Why do countries stock-up on nuclear weapons? Why do cartel agreements break-down? Why do prisoners confess? (Baumol and Blinder (2005), Mankiw (2004)).

Students are usually exposed to games with the players, strategies and payoffs well defined. With the prisoners' dilemma, for example, the students observe how the rules and the payoff matrix leads to the not so intuitively obvious outcome of both players confessing. Yet, they are not given the opportunity to observe how that matrix was constructed and thus do not always develop the competence to analyze unstructured economic problems with game theory.

In this article, we suggest a classroom exercise that can be used in an introductory course where students develop their own strategies, payoffs and rules to a particular scenario. This challenge is memorable, inherently interesting to the students, and humorous. Suppose a professor returns an exam and upon returning to his office realizes that he has not recorded the grades. Can he design a mechanism that will get the students to truthfully report their grades?

This exercise has several useful pedagogical applications. First, it is an active learning exercise and the evidence shows that these exercises increase general interest in the subject, motivation (Holt 1998), and help with material retention (Nilson 2003). Second, it is an example that they can personally relate to much as Trandel (1999) did when he reinforced the notion of a dominant strategy to his students by using an MTV game show. Third, it will enhance their critical thinking and evaluation skills as they discuss different rules and strategies. Fourth, it lends an opportunity to reinforce the idea of a Nash equilibrium.

In the next section we describe the exercise. The third section lists some common questions that students have during the problem-solving phase and our suggested responses. Some sample student solutions and discussion tips are provided in the fourth section. The fifth section provides a generalized solution. The last section contains our conclusion.

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The Exercise

This exercise is easy to implement, involves little preparation, requires minimal materials, and can be conducted in a 50-minute class of any size. We give this exercise to our principles-level students after they have been introduced to the concept of a Nash equilibrium via a simple two-by-two prisoner's dilemma game. Although, we have only done this exercise at the principles level, it is also suitable for upper-level courses.

We recommend breaking the students into groups where each group has four or five students. Each group is presented the following scenario:

> *Professor Gump's Dilemma*
>
> Professor Gump, who is by nature a rather disorganized and scatterbrained educator, finished grading a midterm exam for a section of students. Before going to class to hand back the exam, he calculated the class average but forgot to record the students' individual grades. It was not until after Professor Gump had left the classroom and returned to his office that he realized he had allowed the students to retain their exams, but only had the average grade for all of the students, and not the individual grade for each student.
>
> Your job is to construct a "payoff mechanism" that should encourage each student to tell the truth regarding their grade on the exam, ***under the assumption that the other students are reporting their grades truthfully***. You will have to define the rules, the players, the choices available to the players, and the respective payoffs that they receive when they make their choices. For simplicity, you may assume there are only three students in the class and that their actual exam grades were 60, 80, and 100. Remember, the only information available to Professor Gump is the actual overall average (i.e. 80%) for the class.

We instruct the class that they will have 20 minutes to devise a solution, and after all of the solutions are collected we randomly select three groups to post their answers on the board.

The problem is to design a grade, or payoff, for each student based on the students' reported grades, along with the class average. The words "…under the assumption that the other students are reporting their grades truthfully…" infers that the students are seeking to find a Nash equilibrium where every student has a strategy to truthfully report their grade to the professor.

This is a very challenging problem and there are a lot of questions that you should be prepared to handle up front. In no particular order, a list of commonly asked questions are provided in the next section.

Some Common Student Questions

Since the exercise is unstructured, many of the groups will have questions during the problem-solving phase. We list a few of the most common questions along with our suggested responses.

- ***We are really confused. What are we supposed to do? Can you give a hint?*** We simply remind them that "your task is to develop a mechanism that will get every student to truthfully reveal his or her grade, given the assumption that the other students are truthfully reporting their grades." We guide the process by posing the following question, "What are the choices available to each student? Given these choices and the assumption that the other students are being truthful, develop a payoff mechanism that will encourage every student to tell the truth. Remember that the only thing Professor Gump knows is the class average." We hesitate to say more than this to our students because we want them to discuss the exercise and develop a solution by drawing on their basic knowledge of game theory.
- ***How do we create a payoff matrix when there are more than 2 students?*** A question of this form shows that the students are somewhat fixated on two-player, two strategy games such as those from textbooks. We realize that it may be difficult for them to think in terms of an n-player game so we work with them and suggest that they model this as a game between each student and the rest of the class.

- ***Does Professor Gump reveal the class average to his students?*** Technically, it does not matter if Professor Gump's students know the class average. It will not affect the mechanism that he develops. You may have them consider the following: "If the students know the class average, will this affect their willingness to tell the truth? You need to develop a payoff scheme that encourages them to be truthful whether or not they know the class average." This usually gets the students to refocus on the actual problem at hand.
- ***Can Professor Gump just ask them to return their exams?*** "No, it is not that easy. First, of all, Professor Gump is not that logical. Secondly, there is no guarantee that the students will still have the exams."
- ***What if a student is not present when Professor Gump asks for their grade?*** "For simplicity assume that all students are present. The alternative is much too complicated to consider."

Although not comprehensive, this completes a list of the major questions that our students ask during the problem-solving phase. Generally, most groups construct a solution within twenty minutes although five more minutes may be needed. In the next section we present some of our students' solutions and some proposed discussion points.

Student Solutions and Discussion Tips

After each group representative has placed their solution on the board, we structure our students' presentations by having them answer three questions: (1) Identify the players in this game. (2) What choices does each player have? and (3) How does Professor Gump get his students to truthful report their grades? Below we provide some actual student responses to these questions and we provide some discussion tips for evaluating these responses.

1. ***Identify the players in this game.*** Most groups try to simplify the game by eliminating the person with the 100 and identifying the other two students as the players in this game. We have also had groups model the game as an interaction between Professor Gump and his students. Occasionally, a group produces a rule for truthful reporting that takes all reported grades and the actual class average into consideration. Here are some useful discussion tips and questions:
   - *Why have you made Professor Gump a player in this game? Define his role in this game.* His role is to design the mechanism to solicit the truth. Given the payoff scheme that he develops, it is really the students who are making their decisions based on what they think their fellow classmates will do.
   - *Many of you ignored the student with the perfect exam score because you wanted to simplify this to a 2-player game. However, the game must include all of the students. Consider the following, what if there were more than three students in Professor Gump's class and/or none of the students had a 100?*
2. ***What choices does each player have?*** When the game is modeled as an interaction between the students the most common responses are: 1) "tell the truth" or "lie;" 2) "report a high score" or "report a low score;" and 3) "confess" or "remain silent." The final listed response drives home one of the main objectives of this exercise, and that is to realize that game theory has many interesting applications outside of the prisoner's dilemma. The other two responses raise some interesting discussion questions/points which we address below.
   - *Some of you have stated that each student has only two choices, but they actually have more. What are each student's choices?* In reality a student could "lie" by either reporting a higher or lower score. Furthermore, a student could truthfully report their score or not report a score at all (i.e. remain silent). Most students rule out the possibility of reporting a lower score as being irrational and they also assume that Professor Gump will assign you a zero if you "remain silent."
   - We think it important to state that the 2x2 model can be expanded to include more than two choices and that we will distribute a solution where each student has three alternatives: truthfully report their score, report a higher score, or report a lower score. This solution, which is described in the next section, is given to the students after we review the solutions on the board.
3. ***How does Professor Gump get his students to truthfully report their grades?*** In this portion of the discussion each group reveals the payoff mechanism that they have developed to encourage truthful reporting. Expect to receive a variety of solutions, but overall most of the solutions will either provide an incentive to tell the truth or a penalty for lying. Not all of the solutions will necessarily get the

desired result, but it does get the students thinking in terms of how to get to the Nash equilibrium. Some actual responses follow:

- *"Professor Gump will compare the mean from the reported scores to the mean of the actual scores. If they differ, then everybody gets penalized (e.g. 20 points below the mean or a 0)."* This result will lead to the Nash equilibrium as long as the imposed penalty leaves every student with a lower grade than they actually received on the exam. A 20-point penalty would not deter, for example, the student who scored a 60 from reporting that they got a 100. However, a score of a zero for misreporting would give them the incentive to tell the truth. The fact that all students in the class are penalized when one student lies provides for some lively classroom discussion.
- *"If you don't tell the truth, then the highest grade you will receive is a 59 (i.e. 1 point below the lowest test score). However, if you tell the truth, then you will receive an extra 5 points on your exam."* The problem with this mechanism is that it does not reveal how Professor Gump would detect if they were lying. Furthermore, the score of a 59 implies that Professor Gump knows something about the actual scores when in fact all he knows is the average score.
- *"Professor Gump offers them extra credit if they return their exams. If they do not turn in the exam, then they are penalized (e.g. they are given a 50%)."* This mechanism does not guarantee that all students will return their exam let alone address Professor Gump's concern that he wants them to truthfully report their grades. For example, a student who scored sufficiently low would take the 50% and have no incentive to return their exam. *Is this truthful reporting?*
- Side payments, such as the offering of extra credit, are common. Here's another example. *"Professor Gump tells them that he will give them a certain amount of extra credit if the reported mean equals the actual mean."* The offering of side payments leads to interesting discussion. *Is it really necessary for Professor Gump to offer extra credit in order to solicit truthful reporting?* Most students will recognize that if the penalty is severe enough, then no side payments are needed.

Each of these solutions presents an opportunity to reinforce the idea of a Nash equilibrium where it is in each student's best interest to tell the truth. Although the student at the board may have difficulty demonstrating that their solution is a Nash equilibrium, our experience has found that their classmates are acute in demonstrating why the solution will or will not lead to truthful reporting.

A Generalized Solution

Finally, we distribute a generalized solution, discuss it, and provide some numerical examples to support it. Let s_i represent the i^{th} student's actual grade and r_i represent the grade reported by that same student. The mechanism the students must construct is thus a function as follows:

$$g_i(r_1,...r_n,\bar{s})$$

Where $\bar{s} = \sum_{i=1}^{n} \frac{s_i}{n}$

Thus, a Nash equilibrium requires that for each student *i,*

$$g_i(s_1,...s_i,...s_n,\bar{s}) \geq g_i(s_1,...r_i,...s_n,\bar{s}) \text{ for any } r_i \neq s_i .$$

One mechanism providing such a Nash equilibrium is some form of the following: (a proof is provided in the appendix)

$$g_i = r_i - \begin{cases} 0, & \text{if } \bar{r} \leq \bar{s} \\ k(\bar{r} - \bar{s}), & \text{if } \bar{r} > \bar{s} \end{cases}$$

Where $\bar{r} = \sum_{i=1}^{n} \frac{r_i}{n}$ and $k>n$

The following is an example of a discussion that we have with our students. To see that this grade scheme provides a Nash equilibrium, assume all other students are reporting their grades truthfully and focus on one student. If the student reports a score less than the true score, the student will receive exactly that lower reported score as the grade, and so this is obviously not the student's optimal strategy.

If a student reports a higher grade than is true, the average reported grade for all students is now higher than the actual average grade which is known to Professor Gump. Thus, a penalty is imposed giving that student a lower grade than was earned.

Example: Suppose there are three students, with grades of 100, 80, and 60 for a class average of 80. The professor has assigned a value of k=4 so that if each student has reported accurately, the following values are achieved:

Table 1: All Students Report Their Scores Truthfully

Student, i	Score, s_i	Report, r_i	Grade, g_i
Student 1	100	100	100
Student 2	80	80	80
Student 3	60	60	60

Now assume that student 1 and student 2 have reported their respective scores of 100 and 80 but that student 3 reports some higher grade, say a 90. Then the recorded grades for the students will be as follows:

Table 2: Student 3 Over-Reports His Score

Student, i	Score, s_i	Report, r_i	Grade, g_i
Student 1	100	100	60
Student 2	80	80	40
Student 3	60	90	50

Thus, student 3 would have been better off reporting the actual score earned on the exam, a 60.

In every class at least one student points out that when student 3 lies and reports a score of 90, the other students are also penalized. This provides a great opportunity to demonstrate that this set of strategies is not a Nash equilibrium and a simple example can be used to show that students 1 and 2 would be better off by under-reporting their grades if they knew that student 3 was over-reporting his:

Table 3: Students 1 and 2 Under-Report when Student 3 Over-Reports

Student, i	Score, s_i	Report, r_i	Grade, g_i
Student 1	100	80	80
Student 2	80	70	70
Student 3	60	90	90

If desired, this can then prompt a more elaborate discussion on the meaning of a Nash equilibrium as an outcome to a game in which the participants have no incentive to change their strategy.

Furthermore, player 3's actions in this final numerical example can inevitably lead to several other insights. What if player 3 is not acting rationally because he does not understand that lying will actually lower his grade? This provides an interesting opportunity to discuss the validity of the neoclassical assumption that assumes every agent is rational. Furthermore, in such a case where a player makes an irrational decision, the Nash equilibrium will not be realized. Another possible question, what if player 3 is malicious and gets satisfaction out of lowering everybody's grade? The instructor may wish to point out that these mechanisms do not always work perfectly, however they are structured to solicit the desirable outcome.

Conclusion

The exercise presented in this paper exposes students to the endless possibilities of using economic tools to analyze many interesting things, not just markets. Arguably some of the finest examples of how these tools can be used to study ordinary life can be found in the *New York Times* bestseller, Freakonomics by Levitt and Dubner, where some of the questions answered include: "What do school teachers and sumo wrestlers have in common?" and "Why do drug dealers still live with their moms?"

Game theory is normally used at the introductory level to teach students how to make strategic decisions when faced with a given set of players and a well-defined set of rules, strategies, and a payoff matrix. This exercise enhances students' ability to analyze, evaluate, and synthesize basic principles by encouraging them to develop their own game to solve a specific problem. The inherently interesting aspect of the game engages students to think in strategic terms while providing a vehicle for the instructor to guide that thinking.

References

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APPENDIX

Proof of Generalized Solution

To show that truthful reporting is a Nash Equilibrium, we focus on one student, i, and assume each student $j \neq i$ is reporting truthfully so that $r_j = s_j$, $j \neq i$. If student i reports $r_i < s_i$, the student receives a grade of r_i but could have received higher grade of s_i. Thus, reporting a lower grade than was received is not a grade maximizing strategy for the student, given that the other students are reporting their grades truthfully and so we have shown that reporting a lower grade is not an equilibrium response.

Likewise, suppose the student reports a higher grade, $r_i > s_i$. Assuming all of the other students are reporting truthfully, the grade given to student i is:

$$r_i - \frac{k}{n}\left(\bar{r} - \bar{s}\right) = r_i - \frac{k}{n}\left(r_i - s_i\right)$$

Had the student reported the true grade of s_i, the grade received would be s_i. Comparing these two grades:

$$s_i - \left(r_i - \frac{k}{n}\left(r_i - s_i\right)\right) = \frac{k-n}{n}\left(r_i - s_i\right)$$

However, $k > n$ and $r_i > s_i$, so that the difference is positive and thus, the student is better off reporting truthfully. Thus, we have shown that when the other students are reporting truthfully, student *i*'s best strategy is also to report truthfully so that the mechanism provides for a Nash equilibrium of each student accurately reporting their grades.