A Pedagogical Note on the Derivation of Option Profit Lines

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Abstract

This paper presents a simple graphic method of deriving profit lines for complex option strategies which obviates the need for numerical calculations used in finance textbooks. Having mastered the underlying profit lines associated with simple option and stock investments, students can derive profit lines for complex strategies by following two simple graphic addition rules proposed in this paper. While these rules appear in elementary math books, most finance textbooks either ignore them completely, or apply them only implicitly without explaining their derivation. The result is that the teaching of option profit lines tends to become an unnecessarily arduous exercise.

Introduction

The extensive coverage of derivatives in finance courses heavily emphasizes the use of stock options and the potential gains or losses associated with various option strategies. Textbooks covering these concepts generally present graphic analyses of the gains or losses by deriving the so-called profit lines using rather tedious numerical examples (Johnson and Giacotto, 1995, Mayo, 2003, Reilly and Brown, 2003). These examples typically require that students first calculate the profits or losses using a series of alternative stock prices at expiration of the options and then graph the results. While this arithmetic process is simple enough for basic option strategies, it becomes highly cumbersome with increasing risk of computational error for more complicated option strategies. Furthermore, the process suffers from the shortcoming that students often find it difficult to replicate the results without using numerical tables. Indeed, we have learned through a rather extensive search that some textbooks present more complex strategies by simply providing the graphs without explaining their derivation or providing the underlying numerical values. Learning to derive potential gains and losses is essential to students' training for success in the practice of option trading and of advising investors in the use of options. Indeed a central theme of this paper, based in part on feedback from former students, is that the tools presented herein can facilitate investment strategists' understanding of the benefits and risks associated with various option strategies, and of the underpinnings of these benefits and risks.

The purpose of this note is to provide a simple graphic method of deriving and analyzing option profit lines. The method derives the profit lines for mixed strategies directly by applying two basic graphic addition rules to the profit lines of simple strategies. While these graphic addition rules are readily available in introductory college math books, we have not seen them actually applied in deriving option profit lines. Following these basic rules, the student can begin the analysis by constructing a diagram of the mixed strategy payoffs at alternative expiration date stock prices and then infer the potential gains or losses from the graph.

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Description of the Method of Analysis

To utilize our method, the student must recognize the individual option and stock investments as the building blocks of the analysis. The possibilities of buying or selling calls, puts, and stocks give rise to six building blocks from which first the simple and then the more complicated mixed strategy payoff diagrams can be constructed.

The first step is to derive the profit line associated with a simple strategy involving a single call, put, or stock. Each of these profit lines depicts the relationship between the payoff from the investment and the future price of the underlying stock. The next step is to vertically add the profit lines of simple strategies to obtain the profit lines of mixed strategies. This task is facilitated by dividing the diagram into distinct regions, separated by vertical lines (“walls”) at the exercise prices. Within each region, the profit lines are added using one of the following two simple rules: The first rule stipulates that the addition of a flat line to any other line (flat or slanted) results in a parallel shift of the latter equal to the height (positive or negative) of the flat line. The second rule stipulates that the addition of two divergent and equally sloped slanted lines results in a flat line. (As we show our more advanced students, the sum of two divergent and differently sloped lines is also a slanted line but with a slope that is equal to the algebraic sum of individual slopes). For a reason which becomes apparent in the second example below, it is best to begin the addition process in the region of the diagram where at least one flat line is present.

In the following pages, we demonstrate both the application of these rules to various mixed option strategies and the simplicity they bring to option analysis. To facilitate our exposition, and unless otherwise stated, throughout this paper we assume exercise prices of $100 and premiums of $10 for positions in options, and base prices of $100 for positions in stocks. We further simplify by ignoring all transactions costs so we can analyze the gains and losses solely in terms of the cost bases and market prices of the investments. Finally, we assume all options have the same expiration dates.

To establish the building blocks of the mixed strategies, we begin with the profit lines of the six simple strategies (buy or sell a call, buy or sell a put, buy or sell a stock), as depicted in Figure 1. Here we introduce a convention which will be used to the extent possible throughout the paper: Calls are represented by broken lines, puts by dotted lines, and stocks by solid lines. As the figure shows, the buy (long) and sell (short) positions are simply mirror images of each other. Our classroom experience indicates that students generally master these simple profit lines with little difficulty. It is the mixed strategies that cause most confusion. However, as the following examples demonstrate, our profit line addition rules go a long way in making the learning of options a less arduous experience. We illustrate our technique using five mixed strategies which collectively demonstrate the variations on the technique and its simplicity in deriving profit lines for even complex mixed strategies.
The first mixed option strategy we consider is a long straddle, defined as simultaneous purchases of a call and a put on a given stock, each with the same exercise price. Investors execute this strategy when they expect a significant change in the underlying stock price but are uncertain about its direction.

In Figure 2, the profit lines of the call and put options are represented by the broken and dotted lines, respectively. To add these two lines vertically to derive the profit line of the long straddle, we first draw a vertical line (wall) at the exercise price of the two options ($100). Next, in region I, to the left of the wall, we see that the summation of a slanted line (part of the put profit line) and a negative flat line (part of the call profit line) results in a downward shift of the slanted line, with the extent of the shift given by the size of the call premium ($10). This new slanted line, represented by the heavy, dark line, is the vertical sum of the call and put options, i.e., the profit line of the long straddle, to the left of the exercise price. Likewise, moving to region II, to the right of the wall, we derive the profit line of the long straddle above the exercise price as the heavy line parallel to the broken line of the call. As the figure shows, the overall profit line of the long straddle is V-shaped. Clearly, the buyer of the straddle will earn profits only if the stock price makes a big move in either direction, either below $80 or above $120 from its current level of $100. Otherwise, the buyer will lose all or part of the total premiums paid for the straddle.
Buying a protective put, the investor has a long position in a stock and buys a put to insure against a drop in the stock price. In Figure 3, we begin with the profit lines associated with the stock and the put, respectively. Once again, the “wall” provides two distinct regions, with the combination of horizontal and slanted lines to the right. First, following our rule for combining horizontal and slanted lines, the overall profit line to the right of the wall (region II) is obtained by shifting the slanted line down by the price of the put, as represented by the heavy line in Figure 3. To the left of the wall, applying the rule for addition of two divergent slanted lines, the overall profit line becomes horizontal, as shown by the flat portion of the heavy line.

This example illustrates another important issue regarding the application of our method: We cannot readily begin with two divergent lines, because it is not immediately clear at what height the overall profit line lies. By starting in the rightward region instead, we identify the point at which the overall profit line reaches the wall, and entering the next region, we draw the flat portion of the line as a continuation of the diagonal. In general, we begin in the region where one or both lines are horizontal.
Following this strategy, the investor owns a stock and sells a call on it. This strategy is followed if the investor expects the stock price to change little during the life of the option but to rise in the longer term.

The profit line derivation appears in Figure 4. In region I, to the left of the wall, are a horizontal and a diagonal profit line of the underlying simple strategies, so the overall profit line is diagonal. To the right of the wall, the profit lines run diagonally in opposite directions, so the overall profit line is horizontal.

**Figure 3**
Long Protective Put

**The Short Covered Call**
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**Figure 4**
Short Covered Call

**The Bullish Call Spread**
Following this strategy, the investor buys a call at a given exercise price and sells a call with a higher exercise price. The purchase of the first call reflects the investor’s bullish outlook on the stock, while the sale of the second call reflects a desire to recoup part of the cost of the purchase premium. In Figure 5, with a wall constructed at each of the two exercise prices, there are three regions. In the first region, where both underlying profit lines are horizontal, the resulting total profit line is also horizontal. In region II, the slanted and the flat profit lines result in a slanted total profit line which is a continuation of the total profit line from the first region. Finally, the total profit line in the third region is a horizontal continuation of the total profit line from region II, as the basic profit lines in this region are divergent.

![Bullish Call Spread](image)

**Figure 5**
Bullish Call Spread

**The Long Condor**

Following this strategy, the investor buys both a bullish and a bearish call spread. The condor reflects the expectation that the stock price will remain range bound in the near future. As indicated in Figure 6, which presents the profit lines of the underlying bullish and bearish call spreads, walls are erected at the four exercise prices, resulting in five distinct regions. We begin in the first region, where both underlying profit lines are horizontal, resulting in a flat total profit line. In region II, the slanted and the flat profit lines result in a slanted total profit line which is the continuation of the total profit line from the first region. In the third region, two slanted lines result in an overall profit line which is flat. In the fourth region, an additional pair of slanted and flat profit lines result in another slanted total profit line, which is a continuation of the total profit line from the third region. Finally, the total profit line in the fifth region is a horizontal continuation of the total profit line from the fourth region, as the underlying profit lines in this region are both flat.
This paper provides two simple rules for combining the profit lines of simple strategies and demonstrates how students and practitioners can easily derive the profit lines of more complex strategies. Of course, the user of these procedures must be capable of deriving the profit lines of the underlying simple strategies, but once these simple relations are mastered, analysis of the more complex strategies becomes relatively simple and, in our experience, less subject to error. As noted, we offer the methods of this paper as a learning tool for students, but our students who are professionally employed in the area of options trading have also indicated that they find this method to be of great value in their work.

References

