Teaching MIRR to Improve Comprehension of Investment Performance Evaluation Techniques – Reply to Comment

R. Brian Balyeat¹ and Julie Cagle²

ABSTRACT

Hatem, Johnson, and Yang (2013, this journal) concede that MIRR assumes reinvestment, but claim that neither NPV nor IRR have reinvestment rate assumptions. We show both through simple examples and a discussion of simple versus compound interest that both NPV and IRR require their own reinvestment rate assumption. The confusion arises because these assumptions are designed to have no impact on the respective calculation. By construction, the NPV and IRR methodology assumes that their reinvestment will be project neutral. Thus, these reinvestment assumptions are hidden and do not need to be explicitly taken into account for the calculations.

Introduction

We will address two issues noted in Hatem, Johnston, and Yang (2013, this journal). First, NPV, IRR and MIRR are all calculated using equations based on compound interest, i.e., the reinvestment of interest. While all three assume reinvestment, they may differ with respect to what rate they assume for the reinvestment. We fail to see how it can be a fallacy to say NPV and IRR assume reinvestment when the equations used to calculate them reflect compound interest, yet acknowledge that MIRR, which is also based on equations that assume compound interest, does assume reinvestment. If as Hatem, Johnson, and Yang (2013, this journal) acknowledge, MIRR assumes reinvestment, then so must NPV and IRR.

Consider the following example of a conventional and independent project:

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-100</td>
<td>40</td>
<td>40</td>
<td>40</td>
</tr>
</tbody>
</table>

The IRR of the above project is 9.7%. If we calculate the MIRR for the project assuming a reinvestment rate of 9.7%, the MIRR is also 9.7%. Terminal value is $40(1.097)^2 + 40(1.097) + 40 = 132.02$, and the PV of the outflows is 100. The MIRR is $(132.02/100)^{1/3} - 1 = 9.7\%$. The MIRR and IRR are the same in this case. It is explicit from our MIRR calculation of the terminal value that we were assuming reinvestment at 9.7%, whereas it was hidden in the IRR calculation. In this special case, both the MIRR and the IRR calculation are identical. Additionally, both calculations use time value of money equations that assume compounding and we used the same rate for both of 9.7%. While we could assume different rates for reinvesting other than the 9.7%

¹ Department of Finance, Xavier University
² Department of Finance, Xavier University
for the MIRR calculation, what rate is chosen doesn’t change the common assumption of compounding in the calculations of both MIRR and IRR.

The second issue we disagree with is the rationale for our paper. The rationale is not to teach MIRR, but to improve students’ comprehension of investment performance techniques, of which MIRR is one. Further, it is valid to compare MIRR to NPV and IRR. All three techniques are used to evaluate investment performance. Since MIRR can lead to different decisions than NPV and IRR, then to resolve the conflicts requires the techniques to be compared. Most textbooks after presenting these and other capital budgeting decision criteria compare the techniques in terms of which is/are better indicators of investment performance (e.g., Brigham and Daves (2013, p. 475)).

**Reinvestment Assumption**

Again, the point of our article was how to evaluate investment performance for capital budgeting decisions. We thought it would be helpful to go back to principles of finance and what we teach introductory students. A couple of items immediately jumped to our attention. One is that introductory texts take time to distinguish between the annual percentage rate (APR) and the effective interest rate (EAR). Not only do they distinguish between how the two rates are calculated, but they go on to discuss which to use when quoting a rate according to legislation (APR) and which to use for decision making (EAR). E.g., Ross, Westerfield, and Jordan (2014, p. 142) say, “To compare investments or interest rates, we will always need to convert to effective rates.” Textbooks go on to refer to EAR as the true rate of interest and point out the EAR is the actual rate being paid (Ross, Westerfield, and Jordan (2014, p. 244)) by borrowers. This treatment in principles of finance texts is common across different authors. The primary difference between APR and EAR is that EAR considers intra-year compounding. This suggests that the finance discipline believes the ability to compound, or reinvest, is important for evaluating investment performance, i.e., comparing investment alternatives.

**Bond Context**

In the context of bonds, the YTM is an APR. As suggested by Hatem, Johnston, and Yang (2013) it is a promised yield. If an investment grade bond is compared to another investment alternative presumably this YTM would be converted into an EAR to identify the better alternative. For most corporate bonds with semiannual coupon payments, the EAR would differ from the YTM/APR. In converting to the EAR, the ability to compound/reinvest (earn interest on interest) within the year would be considered. However, if two similar bonds with semiannual coupon payments were compared as is usual the case, the one with the higher YTM/APR would also be the one with the higher EAR. This doesn’t negate that EAR is the best way to compare alternatives, just that the additional calculation to convert to EAR isn’t necessary because the resulting best alternative will be the same alternative reached as comparing YTMs.

We also believe there is a second and more hidden role of reinvestment when comparing investment alternatives. The YTM equation as shown in Cebula and Yang (2008) is derived from the basic time value of money relationship $F=P(1+y)^N$ taught in any introductory finance class. Generally, when we introduce this relationship in a principles class we distinguish between compound interest and simple interest. $F=P(1+y)^N$ is based on the assumption of
compound interest, or said alternatively the ability to reinvest interest. In contrast, the equation
\[ F = P(1 + y)^N \] would be used for simple interest situations. With the assumption of compounding
present value can be found as \[ P = \frac{F}{(1 + y)^N} \] whereas with simple interest \[ P = \frac{F}{1 + yN} \]. Given that
the YTM calculation is the based on former, it would follow that the YTM calculation is based
on the assumption of compound interest. When we solve for today’s equivalent (P) of cash
flows to be received in the future, we are discounting not only the interest that could be earned
on principal during the horizon, but also the interest that could be earned on interest. It follows
that if YTM calculations are using equations that assume compounding, then these YTM
calculation assume the ability to reinvest. Thus, the assumption of reinvestment is *implicit*
in these calculations.

Even in the case of zero coupon bonds when there is only one future cash flow and no
intermediate cash flows to reinvest the equation for valuing the bond or solving for the YTM is
based on semiannual compounding (Emery, Finnerty and Stowe (2011, p. 127)). Emery, et. al.
point out that the semiannual compounding is used to allow comparison with other bonds, but if
you use annual compounding the calculation would provide the Annual Percentage Yield (APY)
which is defined as the effective (true) annual rate of return. To compare the latter to other
alternatives appropriately would mean those alternatives must also be expressed as effective
rates.

**Capital Budgeting Context**

Now we will turn to the context of capital budgeting, which was the focus of our original
article. Both NPV and IRR actually have implicit reinvestment rate/compounding assumptions
(albeit at different rates). The confusion regarding this is the result of the very specific rate
required for reinvestment for both NPV and IRR. The NPV requires that the cash flows are
reinvested at the same rate as the discount rate used in the NPV calculation. Thus, because the
NPV requires (by construction) the use of the discount rate to move cash flows through time
(either by discounting or compounding) the reinvestment of these cash flows can be thought of
as a series of additional zero NPV projects. For example, consider the following four period
project:

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & \\
\mid & \mid & \mid & \mid & \\
-950 & 100 & 100 & 100 & 1100 & \\
\end{array}
\]

The NPV of this project using a discount rate of 10% is $50. Now, if we reinvest the first
$100 cash flow at 10% for two periods, we have the following “new” set of cash flows.

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & \\
\mid & \mid & \mid & \mid & \\
-950 & 100 & 100 & 100 & 1100 & \\
\end{array}
\]

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 & \\
\mid & \mid & \mid & \mid & \\
-100 & +121 & \\
\end{array}
\]

The NPV of this “new” project is the same $50. To achieve this new project the $100
cash flow from the original project’s period one had to be reinvested at the rate of 10% for two
periods to add $121 to the $100 already in period 3 from the original project. While the reinvestment rate assumption is not explicit in the NPV calculation, implicitly any NPV calculation requires that the cash flows can be discounted or compounded (i.e. reinvested) at the prevailing discount rate. However, because the discounting or compounding of the cash flows at the discount rate is in itself a zero NPV project, this does not affect the NPV of the original project. It is because of this that authors say that how cash flows are spent in the future does not affect the value of the project today. By the design of the NPV calculation, the future projects are wealth neutral.

Now, the IRR calculation also has an implicit reinvestment rate assumption. However, unlike the NPV, the reinvestment rate required by the IRR calculation is the IRR itself. Consider a similar example to the previous NPV example. Here we have the following cash flows.

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
\hline
-1000 & 100 & 100 & 100 & 1100
\end{array}
\]

The IRR of the project is exactly 10%. Now, if we reinvest the first $100 cash flow at 10% (the IRR for the project) for two periods, we have the following new set of cash flows.

\[
\begin{array}{cccccc}
0 & 1 & 2 & 3 & 4 \\
\hline
-100 & 100 & 100 & 100 & 1100
\end{array}
\]

The IRR of this new project is also 10%. Thus, the IRR calculation relies on the premise that the intermediate cash flows can be reinvested (or discounted) at the IRR.

The confusion arises because the NPV uses a reinvestment at the rate that does not alter the NPV. Likewise, the IRR requires reinvestment that is at the IRR so that it does not alter the IRR. Thus, for the calculation of both the NPV and the IRR we can ignore these reinvestment projects because they do not change the results. However, this does not mean that they are not there. Both NPV and IRR calculations require that cash flows be discounted at specific rates that provide this neutrality.

Since NPV and IRR calculations use the same time value of money equations as used for YTM calculations, they also implicitly assume reinvestment. In contrast, the MIRR calculation used in Balyeat, Cagle, and Glasgo (2013) requires the manipulation of cash flows by separating those that are part of the investment base for a project from those that are available to reinvest elsewhere in the firm. Those that make up the investment base are then discounted to the present, while those available elsewhere in the firm are compounded until the termination of the project. Thus, it is very obvious when calculating the MIRR this way that reinvestment is occurring and what rate is being assumed. By this we mean the reinvestment assumption is explicit and not hidden.
Conclusion

Our examples show that when the firm uses the NPV criteria, it implies that the firm is indifferent to exchanging cash flows across time periods at the WACC. Likewise, when using the IRR criteria, the firm is willing to exchange cash flows across time at the IRR. Whether you call this “exchange” reinvestment or not is semantics, but use of the term would seem appropriate with students that have been taught the importance of earning interest on interest. The main premise of our original paper is that by teaching the MIRR one reinforces the primacy of the NPV criteria by showing students that being willing to exchange cash flows across time at a rate other than the WACC (e.g. at the IRR) is inconsistent with maximizing shareholder wealth.
References


